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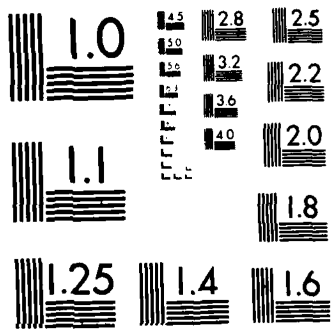

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EFFECT OF NOISE IN THE THREE-PARAMETER LOGISTIC MODEL

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EFFECT OF NOISE IN THE THREE-PARAMETER

LOGISTIC MODEL

ABSTRACT

In the preceding research report, ONR/RR-82-1 (Information Loss Caused by Noise in Models for Dichotomous Items), observations were made on the effect of noise accommodated in different types of models on the dichotomous response level. In the present paper, focus is put upon the three-parameter logistic model, which is widely used among researchers. An emphasis is put upon the speed of convergence to the normality of the conditional distribution of the maximum likelihood estimate, given a specific ability level.

The research was conducted at the principal investigator's laboratory, 405 Austin Peay Hall, Department of Psychology, University of Tennessee, Knoxville, Tennessee. Those who worked for her as assistants include Paul S. Changas, Charles T. McCarter, Vicki Newton, Donald Reece Danna, Cornelia Chapman, Hossein H. Kord, Donna Reynolds, Nancy Cossentine, and Philip S. Livingston.

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I Introduction

This is a continuation of the previous research report, which is entitled, "Information Loss Caused by Noise in Models for Dichotomous Items." In the present paper, we will focus our attention upon the three-parameter logistic model, which is widely used among researchers as a model for the multiple-choice test item in comparison with the normal ogive model.

Throughout this paper, we shall solely consider the unidimensional latent space. Let θ denote the latent trait which assumes any real number. In dealing with the multiple-choice test item, there exist two distinct standpoints: 1) to treat the item as a dichotomous item, classifying the correct answer into one category and all the other alternative answers into the other, and 2) to treat it as a polychotomous item by acknowledging each alternative as an individual resource of information. In the former case, it is most common to define the binary item score, u_g, for item g and assign $u_g = 1$ to the correct answer and $u_g = 0$ to all the other alternative, incorrect answers. If we accept the knowledge or random guessing principle, i.e., that the examinee either knows the answer or guesses randomly, the three-parameter normal ogive, or logistic, model must be an appropriate model. An advantage of the model may be its simplicity. Two main disadvantages are, however, that: 1) in many practical situations, the knowledge or random guessing principle is not applicable, and 2) because of the noise caused by random guessing, we must do without certain mathematical properties which otherwise we could enjoy. If we take the second standpoint, we must

estimate the operating characteristic of each of the incorrect alternative answers, which sometimes are called distractors, in addition to the one for the correct answer. The operating characteristic of a distractor is called the plausibility function. A family of models for the multiple-choice test item, which takes both the characteristics of each distractor and noise caused by random guessing, has been proposed (Samejima, RR-79-4, Final Report).

In comparison with the first standpoint, there is no question that the second standpoint is better, in the sense that each test item will provide us with a greater amount of information, which leads to the more efficient estimation of the examinee's ability, or latent trait. Although it requires more mathematical sophistication in dealing with it, it may be time that researchers switch to the second standpoint and enjoy its benefits. In estimating the plausibility functions of distractors, methods and approaches for estimating the operating characteristics of discrete item responses, such as Levine's (Levine, 1981) and Samejima's (Samejima, 1977a, RR-77-1, RR-78-1 to RR-78-6, RR-80-2, RR-80-4, RR-81-3, Final Report), will be useful. It has been found, amazingly, that many existing multiple-choice test items have informative distractors. In the future, however, it is desirable to modify the guidelines of test construction to encourage test developers to include more informative distractors, and to show how and with what principle they should do that.

At present, unfortunately, this second tide is yet to come. Researchers use the three-parameter logistic model even if their data contradict the knowledge or random guessing principle. They claim that

the function can still be an approximation to the operating characteristic of the correct answer, regardless of the principle it may follow.

In the present paper, we will not question the adequacy of the three-parameter logistic model further than we already have. We will put ourselves in an assumption that, no matter what, we must use the three-parameter logistic model in a given situation. To begin with, we shall compare the three-parameter logistic model with the normal ogive model and find out, quantitatively, how much the three-parameter logistic model has to lose and, qualitatively, what kinds of deficiencies the model has because of the noise caused by random guessing.

II <u>General Characteristics of the Three-Parameter Logistic Model</u>

Let $P_g(\theta)$ be the operating characteristic of the correct answer to item g, or the item characteristic function. This function is the conditional probability, given θ, with which the subject answers item g correctly. Since this conditional probability also equals the mean of the conditional distribution of the binary item score u_g, given θ, $P_g(\theta)$ is also the regression of the binary item score u_g on ability θ. Three-parameter logistic model is defined by the item characteristic function such that

$$(2.1) \qquad P_g(\theta) = c_g + (1-c_g)\,\Psi_g(\theta) \quad ,$$

where c_g is a constant which equals the reciprocal of the number of the alternatives attached to the multiple-choice test item, which is called

the guessing parameter, and $\Psi_g(\theta)$ is given by

$$(2.2) \qquad \Psi_g(\theta) = [1 + \exp\{-Da_g(\theta-b_g)\}]^{-1} \quad .$$

This function $\Psi_g(\theta)$ itself is the item characteristic function in the (two-parameter) logistic model. The two item parameters, a_g and b_g, in (2.2) are called the item discrimination parameter and the item difficulty parameter, respectively. The item discrimination parameter assumes any finite, positive value, and the item difficulty parameter takes on any finite, real number. The seemingly redundant constant, D, is a scaling factor, which adjusts the value of the discrimination parameter, a_g. When we set this scaling factor equal to 1.7, the same set of two item parameters, a_g and b_g, provides us with $\Psi_g(\theta)$ which is very close to the corresponding item characteristic function, $\Psi_g(\theta)$, in the normal ogive model (Birnbaum, 1968), which is defined by

$$(2.3) \qquad \Phi_g(\theta) = (2\pi)^{-1/2} \int_{-\infty}^{a_g(\theta-b_g)} e^{-u^2/2} du \quad .$$

For this reason, the logistic model was originally developed as a substitute for the normal ogive model. The former provides us with a sufficient statistic, $t(V)$, which is given by

$$(2.4) \qquad t(V) = \sum_{g=1}^{n} a_g u_g \quad ,$$

for the response pattern,

$$(2.5) \qquad V = (u_1, u_2, \cdots, u_g, \cdots, u_n)' \quad ,$$

where n is the number of items in the test. The maximum likelihood estimate, $\hat{\theta}_V$. of ability θ is given by the solution of

$$(2.6) \qquad t(V) = \sum_{g=1}^{n} a_g \Psi_g(\theta)$$

for the (two-parameter) logistic model.

It has been reported (e.g., Lord, 1968) that an unrestricted estimation of the three parameters in the three-parameter logistic model provides us with the estimated guessing parameter which is substantially different from the reciprocal of the number of the alternatives attached to the multiple-choice item, and very often the value is less. Note that this fact itself is an invalidation of the model. Many researchers stick to the model, however, as a simple numerical approximation to some unknown item characteristic function, which exists behind their empirical data and whose formula and rationale are hidden. For this reason, this third parameter c_g is sometimes called pseudo-guessing parameter. This interpretation of the three-parameter logistic model casts some doubt, however. It has been shown (Samejima, RR-79-4; April 1980) that, if our test item follows a model in the new family of models for the multiple-choice test item, the operating characteristic of the correct answer, or item characteristic function, tends to have a non-monotonic form. Figure 2-1 presents several typical item characteristic functions in Models A, B

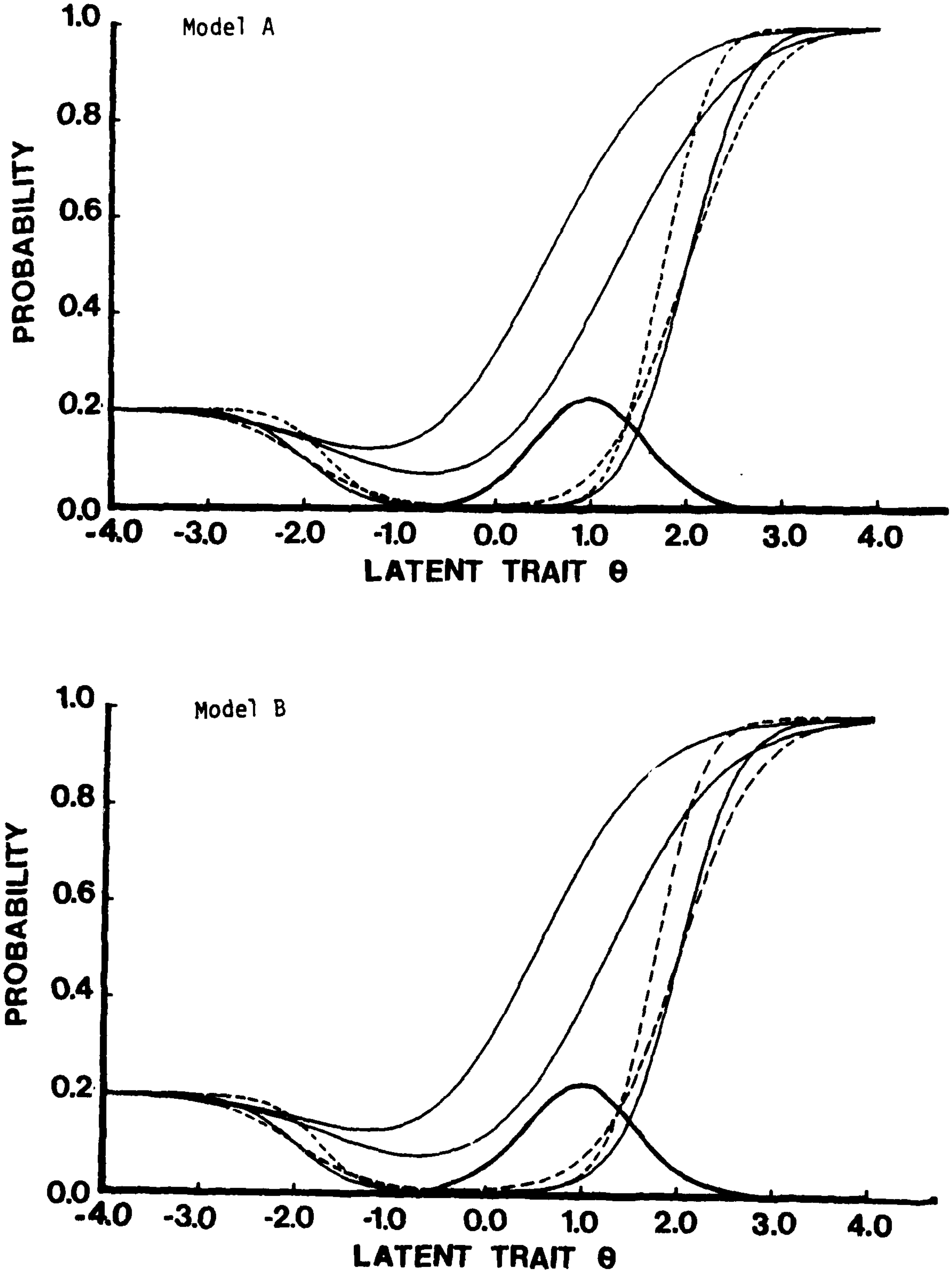


FIGURE 2-1

Typical Operating Characteristics of the Correct Answer in Models A, B and C. Ability Distribution of a Group of Hypothetical Examinees is Also Drawn by a Thick, Solid Line.

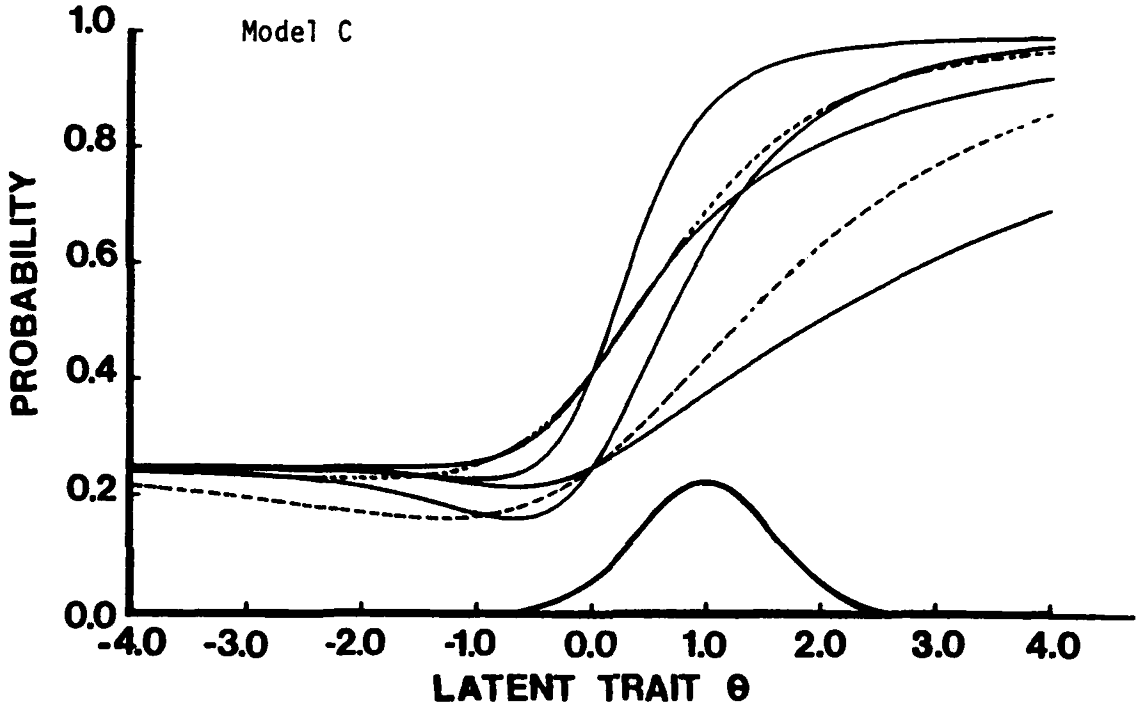


FIGURE 2-1 (Continued)

and C, respectively, which were taken from one of :r previous works (Samejima, RR-79-4). We can see in this figure that each operating characteristic of the correct answer decreases in θ up to a certain level, and then starts increasing. If, for instance, our calibration data have been collected for a group of subjects whose ability distribution follows the density drawn by a thick, solid line in each graph of Figure 2-1, and if, nonetheless, we assume the three-parameter logistic model for our items, then the estimates of the parameters c_g will be less than the reciprocal of the number of the alternatives. For the purpose of illustration, Figure 2-2 presents one of the curves of Model A with $a_g = 1.00$, $b_{x_g} = -1.50$, -1.00 , -0.50 , 0.00 , 0.50 , which was taken from the first graph of Figure 2-1, by a solid line, and the item characteristic function in the three-parameter logistic model with $a_g = 1.00$, $b_g = 0.60$ and a pseudo-guessing parameter 0.05 by a dotted line. Comparison of these two curves in Figure 2-2 suggests that, for the interval of θ where most of the subjects of our calibration data are located, these two item characteristic functions are practically identical. And yet the danger of accepting the three-parameter logistic model as the approximation for Model A is obvious, for the discrepancies are substantial outside this interval of θ . If, for instance, the estimated item characteristic function in the three-parameter logistic model thus calibrated is applied for data collected for another group of subjects whose ability distribution is shifted to a lower side of θ , then we will have a serious problem in analyzing our data because of these discrepancies. This fact also implies the danger of using a single set of

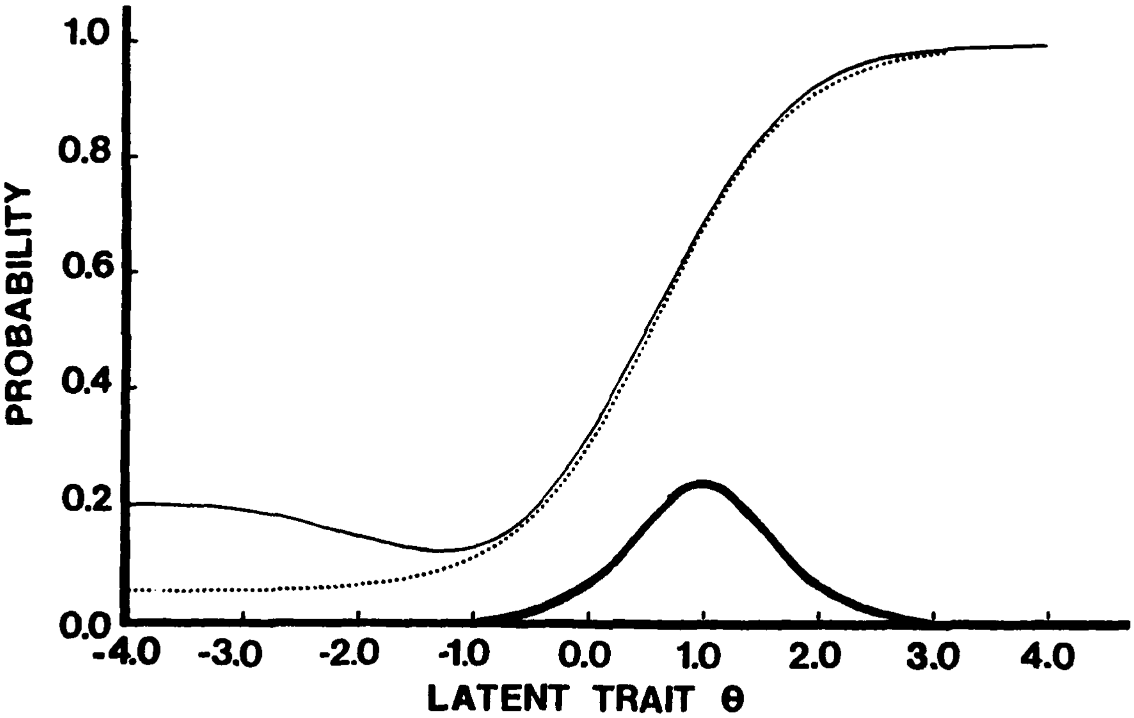


FIGURE 2-2

The Item Characteristic Function of an Item Following Model A with the Parameters $a_g = 1.00$, $b_{x_g} = -1.50$, -1.00 , -0.50 , 0.00 and 0.50 for $x_g = 1,2,3,4,5$ (Solid Line), and the One Following the Three-Parameter Logistic Model with $a_g = 1.00$, $b_g = 0.60$ and $c_g = 0.05$ (Dotted Line).

data in model validation. It should be kept in mind especially when our model fails to provide us with a sound rationale, as the three-parameter logistic model with a pseudo-guessing parameter does.

It has been pointed out (Samejima, 1972, 1973) that, unlike the (two-parameter) normal ogive and logistic models, the three-parameter logistic model does not provide us with a unique modal point for the likelihood function of every possible response pattern. We can write for the basic function (Samejima, 1969, 1972), $A_{u_g}(\theta)$, in the three-parameter logistic model,

$$(2.7) \qquad A_{u_g}(\theta) \begin{cases} = -Da_g \Psi_g(\theta) & u_g = 0 \\ = [(1-c_g)\, Da_g\, \Psi_g(\theta)\{1-\Psi_g(\theta)\}]\, [c_g+(1-c_g)\Psi_g(\theta)]^{-1} & u_g = 1 \end{cases} .$$

From (2.7) it is obvious that the basic function is not strictly decreasing in θ for $u_g = 1$, although it is for $u_g = 0$. This leads to the fact that, while either in the normal ogive model or in the logistic model the item response information function (Samejima, 1972), $I_{u_g}(\theta)$, assumes positive values throughout the entire range of ability θ for both $u_g = 0$ and $u_g = 1$, in the three-parameter logistic model there is an interval of θ where $I_{u_g}(\theta)$ assumes negative values for $u_g = 1$. This interval is $(-\infty , \theta_g)$, where θ_g is given by

$$(2.8) \qquad \theta_g = (2Da_g)^{-1} \log c_g + b_g \quad .$$

Several observations were made for the item response information function

in the three-parameter logistic model, which was used as an example of the Type B model (Samejima, ONR/RR-82-1).

III Loss of Accuracy in Ability Estimation Caused by Random Guessing

In this section, we shall observe the loss of accuracy in estimating the examinee's ability caused by random guessing, by comparing the local standard errors of estimation of different hypothetical tests which follow the normal ogive model and the three-parameter logistic model with $c_g = 0.20$ and $c_g = 0.25$, respectively. In so doing, we shall use hypothetical tests of equivalent items, or items having identical item characteristic functions. It was pointed out in the preceding section that the three-parameter logistic model does not assure a unique maximum for the likelihood function of every possible response pattern, and the item response information function for $u_g = 1$ assumes negative values for the interval, $(-\infty, \theta_g)$. Thus it may be meaningless to discuss the standard error of estimation when single maximum likelihood estimates may not exist for some response patterns. If a test consists of equivalent items, however, a unique maximum likelihood estimate always exists for every response pattern, regardless of the model the items follow. In fact, the simple test score t , which is the sum total of the binary item scores, is a simple sufficient statistic for the response pattern V , and the maximum likelihood estimate, $\hat{\theta}_t$, is obtained as the solution of

$$(3.1) \qquad (t/n) = P_g(\theta) \quad .$$

Note, however, that, if the item follows the three-parameter logistic model, or any other model of Type B, (3.1) will not have a solution if the relative test score, (t/n), is less than c_g. In such a case, the maximum likelihood estimate is negative infinity (cf. Samejima, ONR/RR-82-1).

In general, for a binary item g, the item response information function, $I_{u_g}(\theta)$, is defined by

$$(3.2)\qquad I_{u_g}(\theta)\begin{cases} = -\dfrac{\partial^2}{\partial\theta^2}\log Q_g(\theta) & u_g = 0 \\ = -\dfrac{\partial^2}{\partial\theta^2}\log P_g(\theta) & u_g = 1 \end{cases},$$

where $P_g(\theta)$ and $Q_g(\theta)$ $[= 1-P_g(\theta)]$ are the operating characteristics of the correct and incorrect answers for item g, respectively. In the three-parameter logistic model, $P_g(\theta)$ is given by (2.1) with (2.2) substituting for $\Psi_g(\theta)$, and, in the normal ogive model, it is replaced by the right hand side of (2.3). The item information function, $I_g(\theta)$, is the conditional expectation of the item response information function, $I_{u_g}(\theta)$, given θ, and for a binary item g we obtain

$$(3.3)\qquad I_g(\theta) = E[I_{u_g}(\theta)|\theta] = [\frac{\partial}{\partial\theta}P_g(\theta)]^2\,[P_g(\theta)Q_g(\theta)]^{-1}\ .$$

.t has been shown (Samejima, RR-79-1, ONR/RR-82-1) that there exists some stancy for the amount of information given by a binary item, and, in particular, models of Type A which provide us with strictly increasing item characteristic functions with zero and unity as the two asymptotes, and to which the normal ogive model belongs, the area under the curve of

the square root of the item information function equals π . It has also been pointed out (Samejima, ONR/RR-82-1) that, for models of Type B, whose item characteristic functions are given by (2.1), and to which the three-parameter logistic model belongs, this area, Q , is given by

$$(3.4) \qquad Q = \pi - 2\tan^{-1}[c_g/(1-c_g)]^{1/2} \quad .$$

When $c_g = 0.20$, Q equals, approximately, 0.705π , and when $c_g = 0.25$, it is approximately 0.667π .

When a test consists of only one item, the item information function $I_g(\theta)$ equals the test information function $I(\theta)$, and the characteristics of the square root of the item information function apply directly for the square root of the item information function. In practice, however, it is a highly unlikely case, and there usually are more than one binary item in a test. The response pattern information function, $I_V(\theta)$, is defined by

$$(3.5) \qquad I_V(\theta) = -\frac{\partial^2}{\partial\theta^2}\log P_V(\theta) \quad ,$$

where $P_V(\theta)$ is the operating characteristic, or the conditional probability, given θ , of the response pattern V . When the conditional independence of the item score distributions, given θ , holds, this operating characteristic is given as the product of the operating characteristics of u_g which belong to the response pattern V . The test information function is defined as the conditional expectation of the

response pattern information function, given θ , and thus we can write

$$(3.6) \qquad I(\theta) = E[I_V(\theta)|\theta] = \sum_V I(\theta)\, P_V(\theta) \quad .$$

From (3.6), following through some mathematics, we obtain

$$(3.7) \qquad I(\theta) = \sum_{g=1}^{n} I_g(\theta) \quad ,$$

provided that the conditional independence of the item score distribution holds.

We notice that, if the term under the summation on the right hand side of (3.7) were the square root of the item information function, instead of the item information function itself, then there would be a similar constancy for the amount of test information as there is for the amount of item information. As it is, however, there is no such constancy for the square root of the test information function, and, therefore, different combinations of items will provide us with different values for the area under the curve of the square root of the test information function.

When a test consists of n equivalent items, however, there exists a similar kind of constancy for the square root of the test information function, for from (3.7) we can write

$$(3.8) \qquad [I(\theta)]^{1/2} = n^{1/2}[I_g(\theta)]^{1/2} \quad ,$$

where $I_g(\theta)$ is the common item information function. This indicates that, when a test consists of n equivalent binary items following the normal ogive model, the area under the square root of the test information function equals $n^{1/2}\pi$, regardless of the common item parameters, a_g and b_g, of those equivalent items. (3.8) also implies that, if a test has n equivalent binary items following the three-parameter logistic model with $c_g = 0.20$, then the area under the curve of the square root of the test information function approximately equals $0.705\, n^{1/2}\pi$, and with $c_g = 0.25$ it equals approximately $0.667\, n^{1/2}\pi$. Figure 3-1 illustrates the square roots of test information functions in the normal ogive model and in the three-parameter logistic model with $c_g = 0.20$ and $c_g = 0.25$ with the other common parameters $a_g = 0.50$ and $b_g = 0.00$, by a dashed line and two solid lines, respectively, for $n = 10, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200$. A similar comparison with respect to the three test information functions was made and is shown in Appendix, as Figure A-1.

We notice in those graphs of Figure 3-1 that the two curves for the three-parameter logistic model are much closer to each other, compared with their relationship with the curve for the normal ogive model. The values of θ_g, which were obtained by (2.8), turned out to be -0.9467282900048181 and -0.8154673627399685 for $c_g = 0.20$ and $c_g = 0.25$, respectively, and these values are also shown in Figure 3-1. It is observed that the distances between the curve for the normal ogive model and each of the two curves for the three-parameter logistic model are rapidly enhanced as θ departs from θ_g in the negative direction.

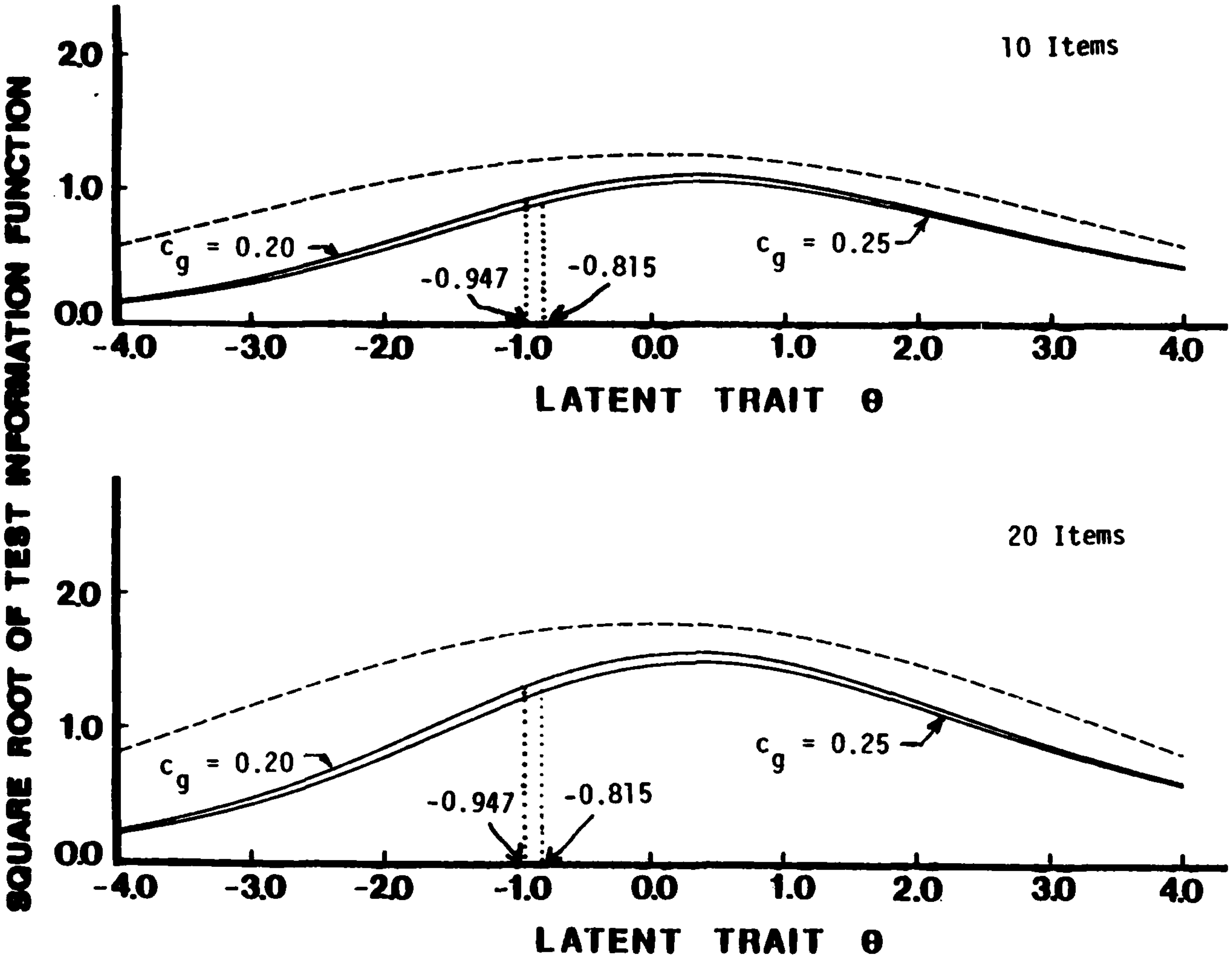
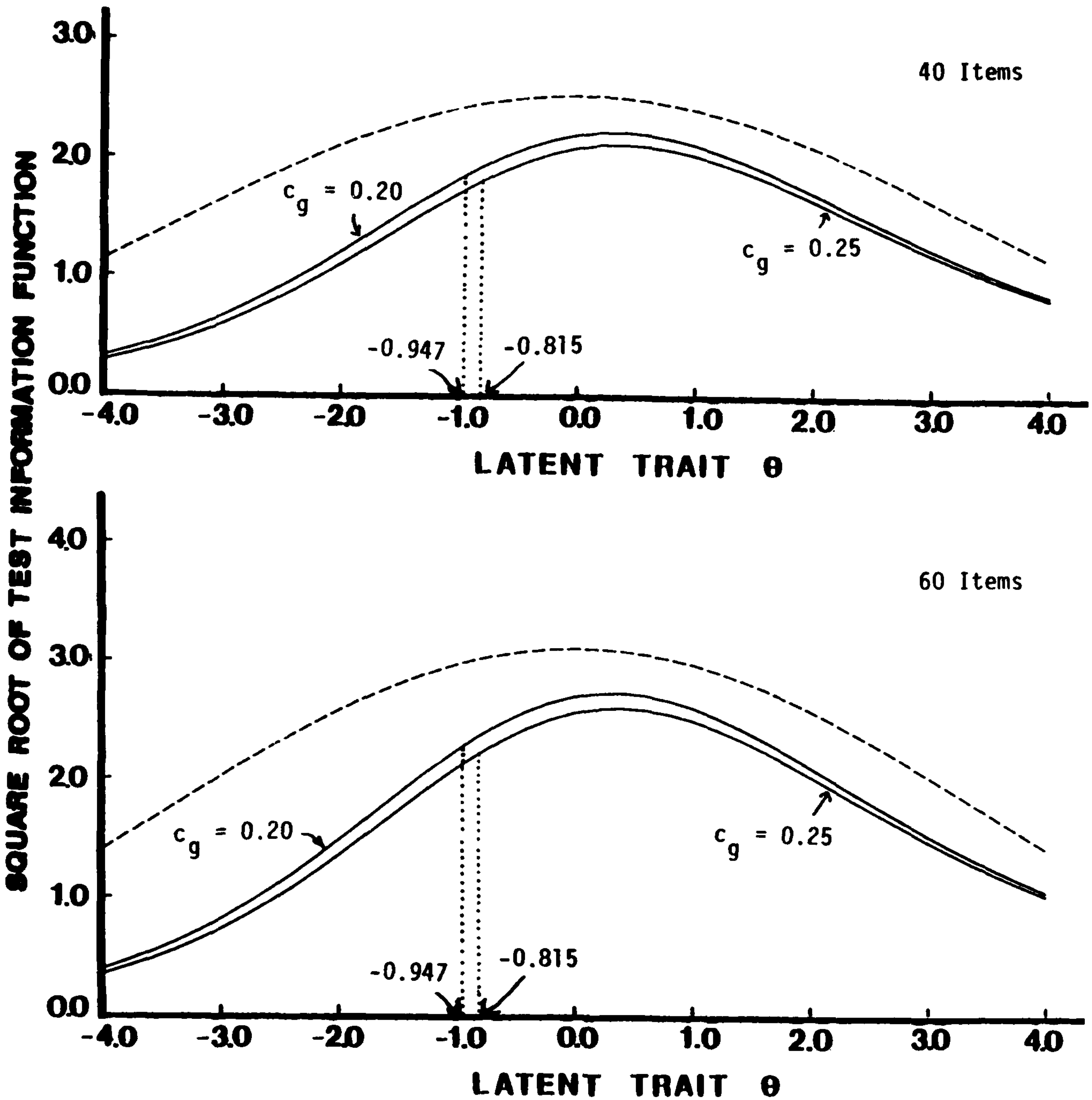


FIGURE 3-1

Square Roots of Test Information Functions of Each of Eleven Tests of Equivalent Items, in the Normal Ogive Model (Dashed Line), and in the Three-Parameter Logistic Model (Solid Lines) with $c_g = 0.20$ and $c_g = 0.25$. The Two Values of the Common θ_g Are Shown.



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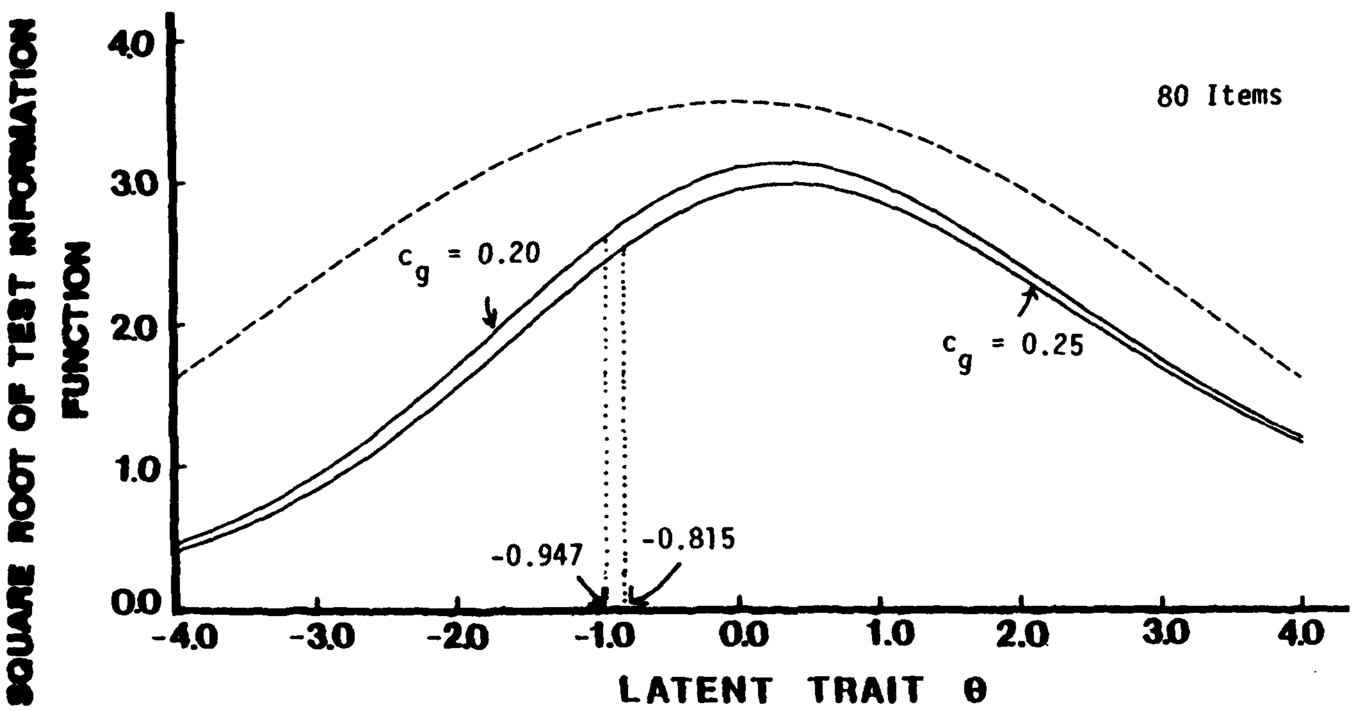

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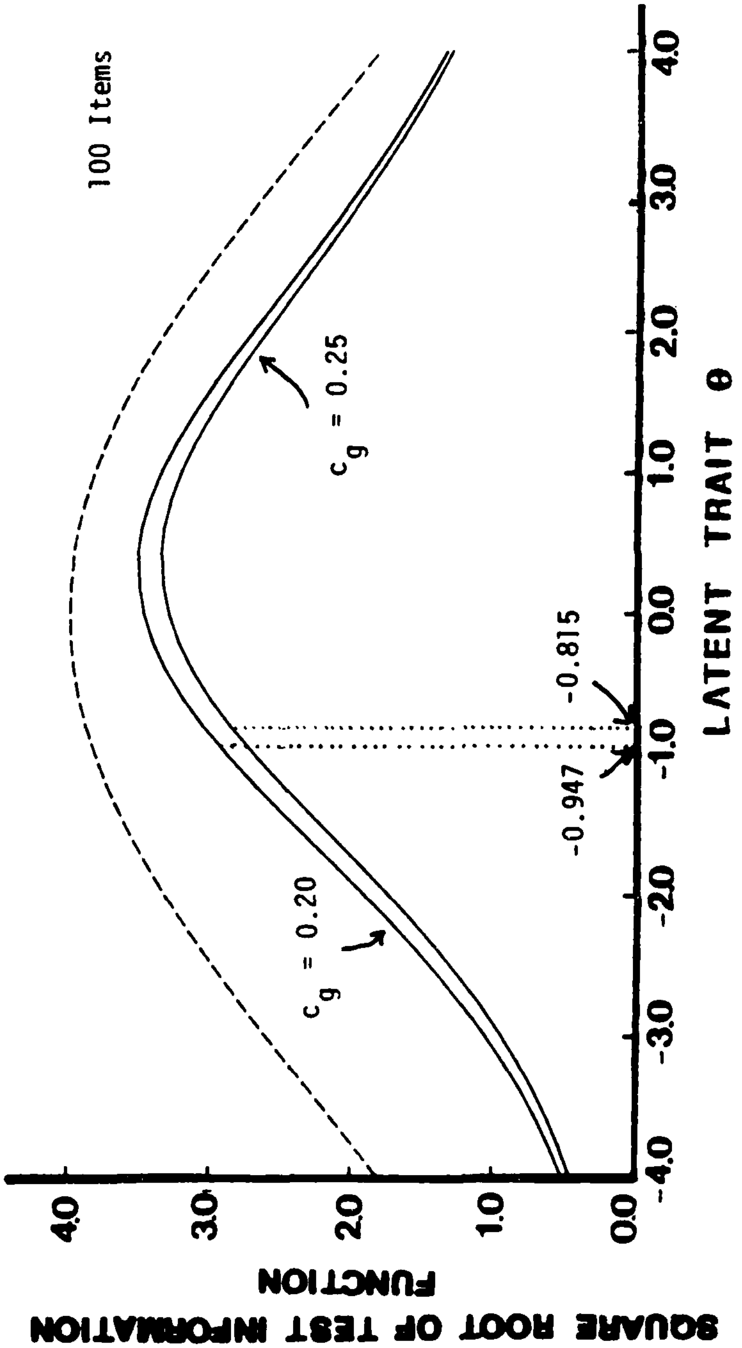

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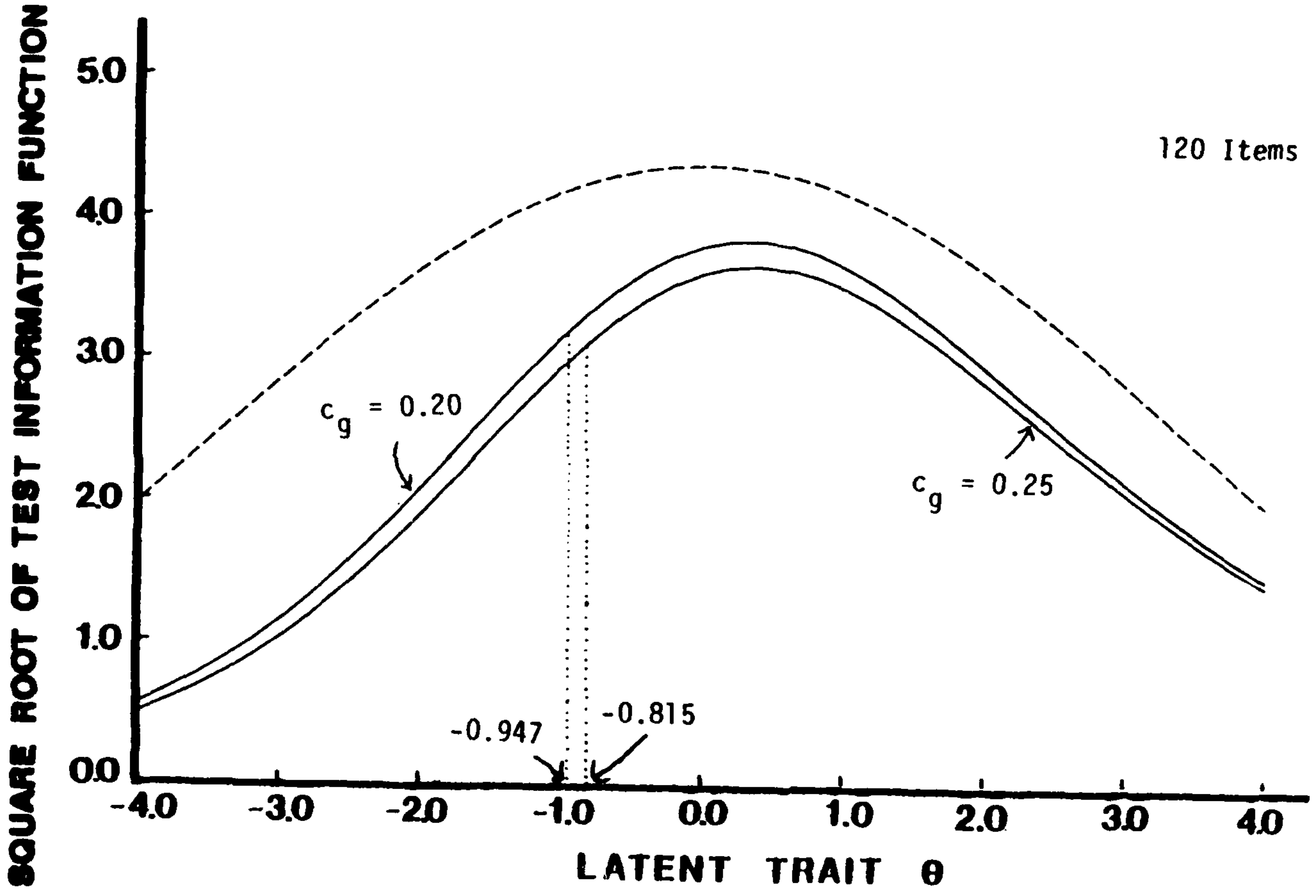

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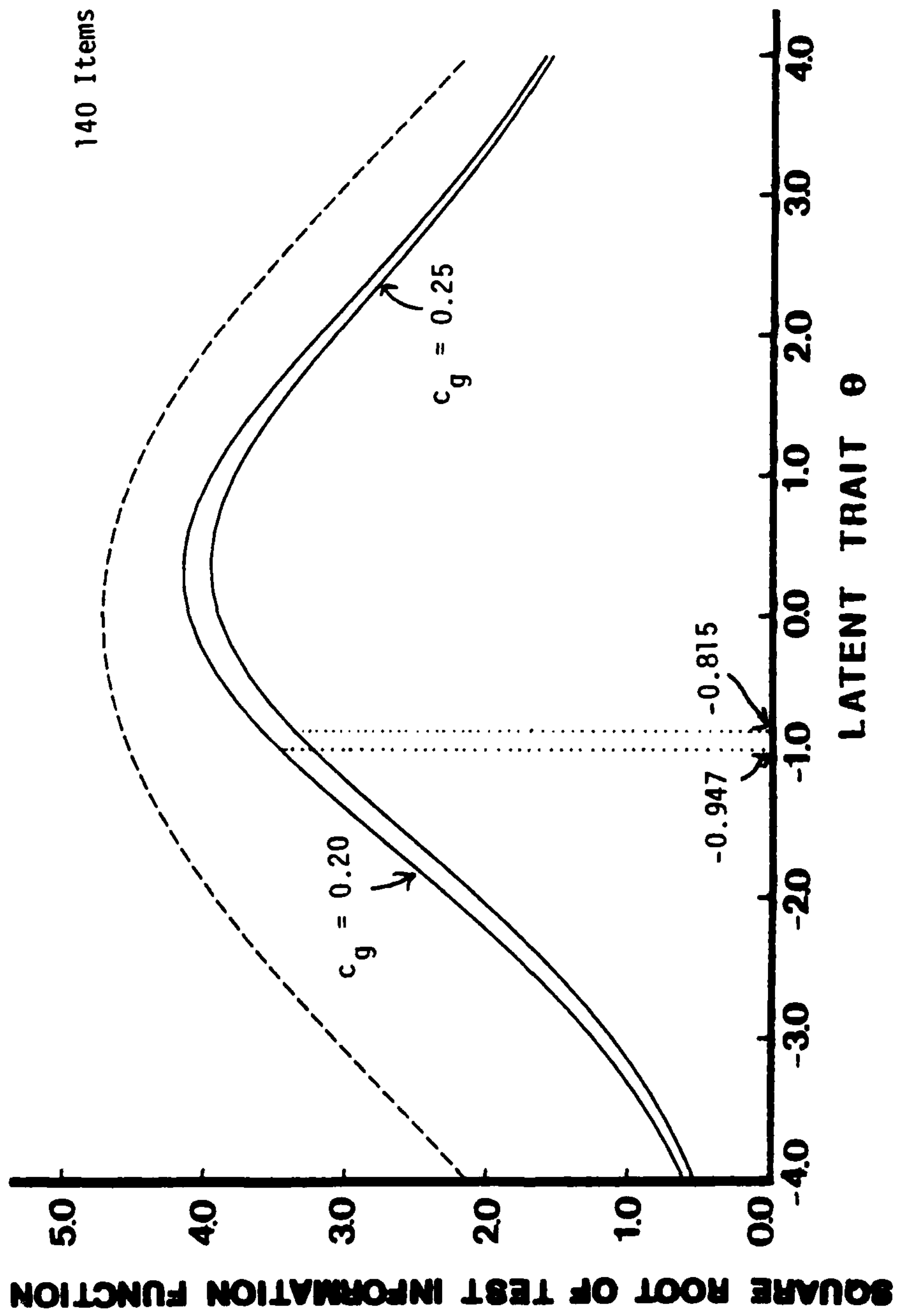

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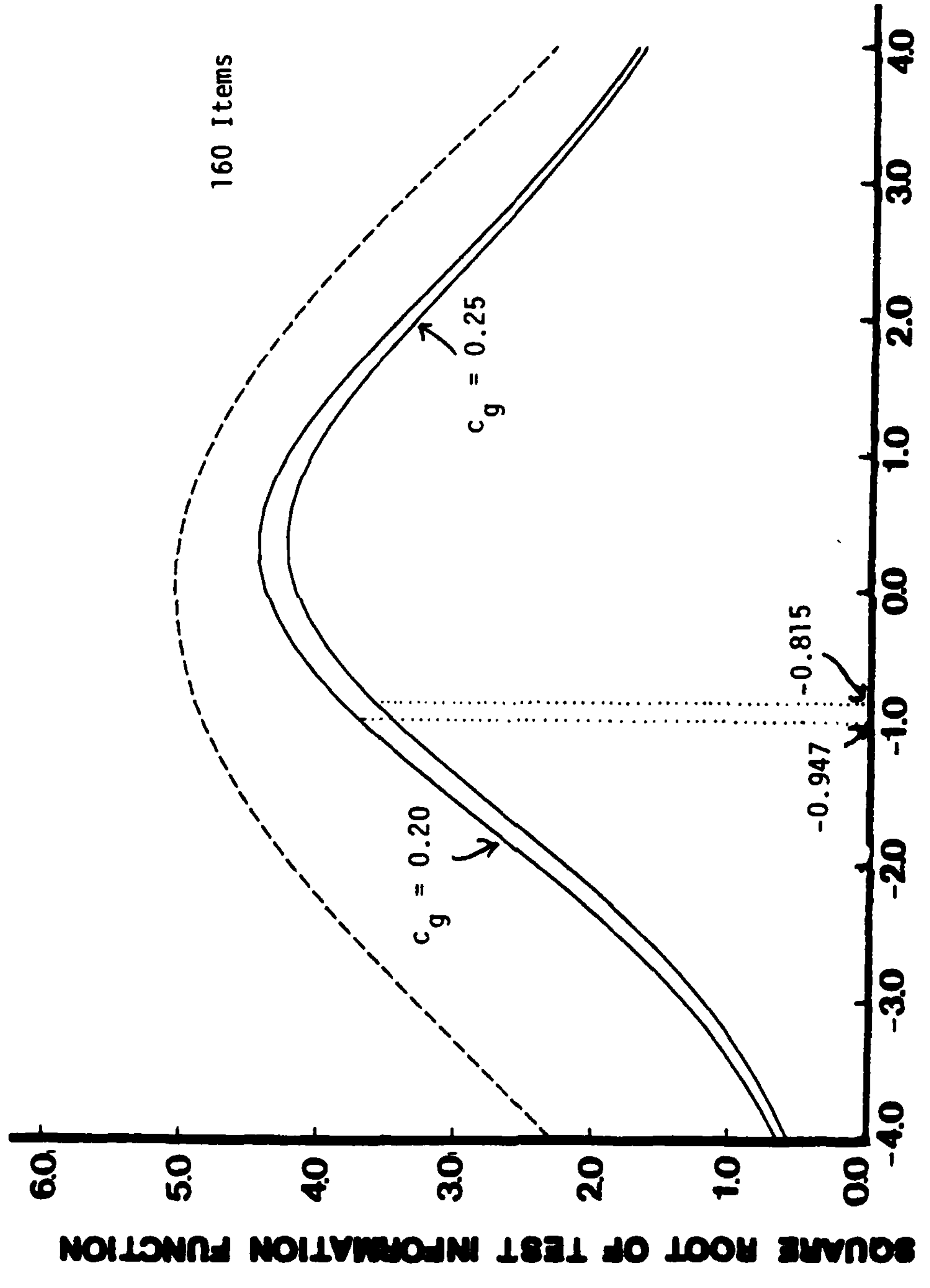

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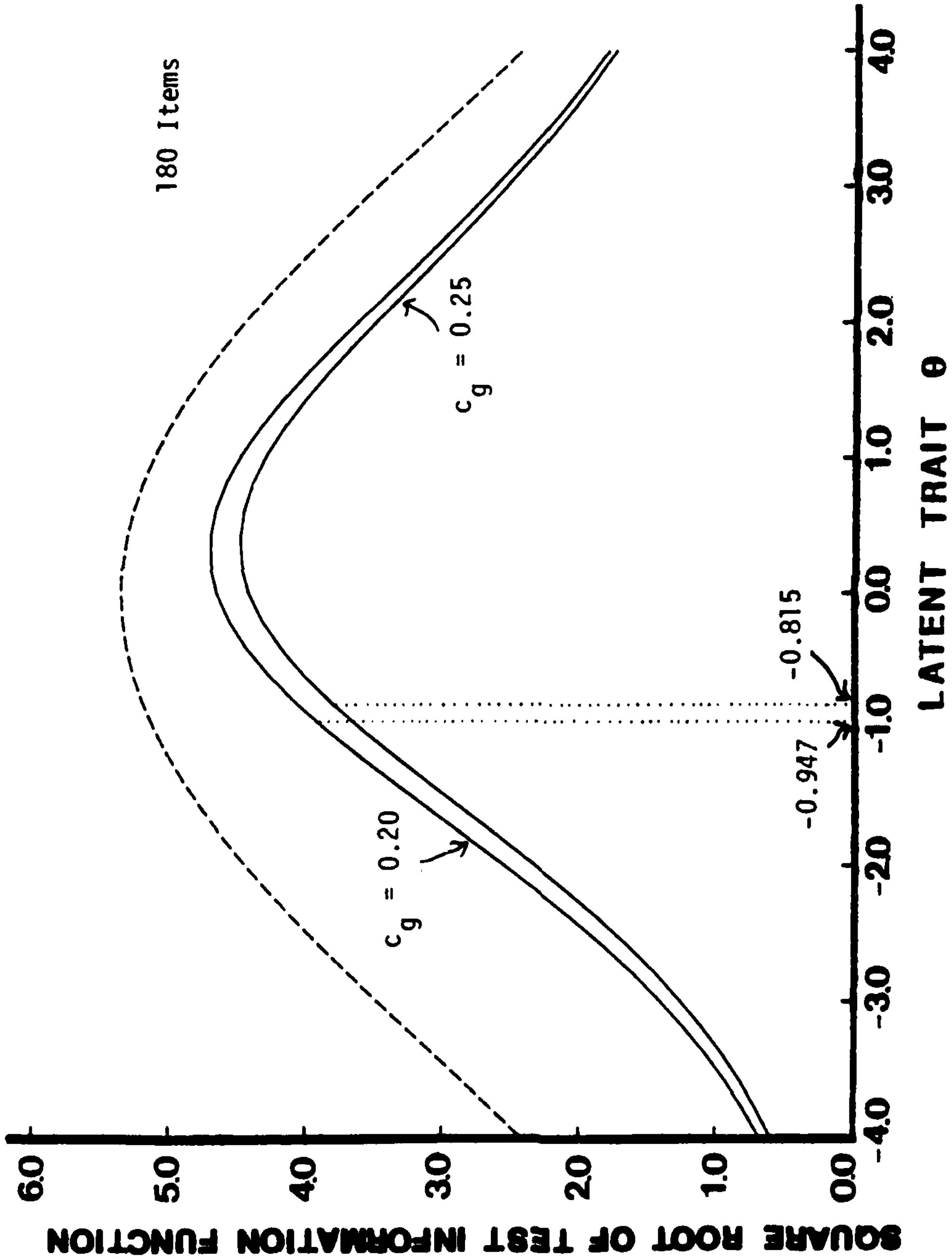

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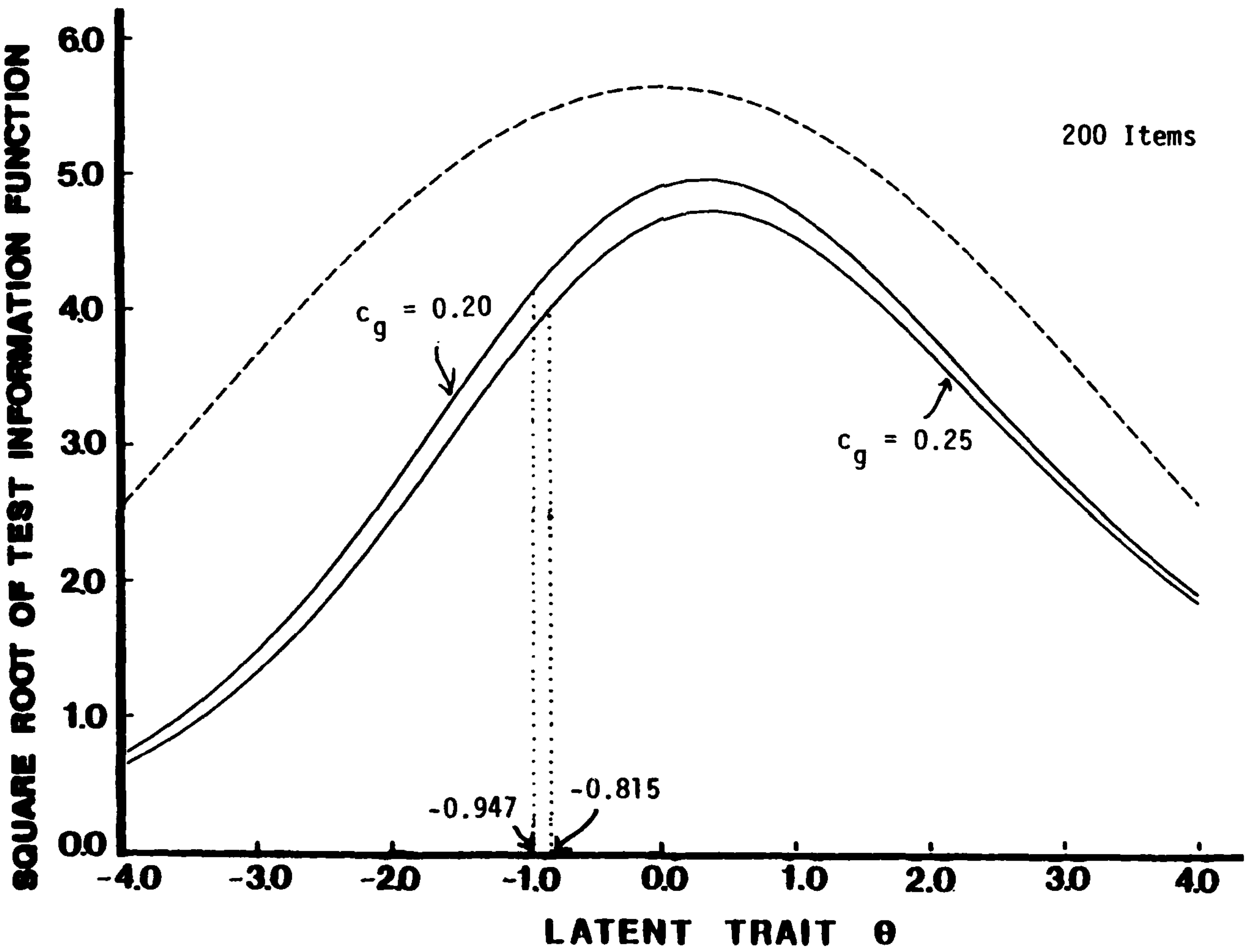


FIGURE 3-1 (Continued)

We notice that those graphs of three curves can be used for other sets of the parameters a_g and b_g , as long as we keep the values of c_g as they are. In so doing, we need to change the scale values shown on the abscissa and on the ordinate. If, for instance, we wish to use the graphs for $a_g = 1.00$ instead of $a_g = 0.50$ without changing the value of b_g $(= 0.00)$, then we must change the numbers on the abscissa of Figure 3-1 to their half values, and also make those numbers on the ordinate twice as large as the original values. If we wish to use Figure A-1 in Appendix for the same purpose, then we will have to change the numbers on the abscissa in the same way as we did for Figure 3-1 and make the numbers on the ordinate four times as large as the original values.

By virtue of the asymptotic normality of the conditional distribution of the maximum likelihood estimate $\hat{\theta}_V$, given θ (e.g., Kendall and Stuart, 1961, Samejima, 1975), the asymptotic regression of $\hat{\theta}_V$ on θ equals θ itself, i.e., the maximum likelihood estimate $_V$ is asymptotically conditionally unbiased, and the conditional distribution is asymptotically normal with θ and $[I(\theta)]^{1/2}$ as its two parameters. It has been shown (Samejima, 1977a, 1977b, Final Report) that even for a relatively small number of items and a relatively small amount of test information this asymptotic property can be used as a good approximation to the conditional distribution of $\hat{\theta}_V$. Figure 3-2 presents the interval of $\hat{\theta}_V$ which was obtained by using θ as the regression of $\hat{\theta}_V$ on θ and $[I(\theta)]^{1/2}$ as the standard error of estimation, which is plotted both above and below the regression line. In this figure, the two curves are drawn by solid lines for the normal ogive model, and they are drawn by

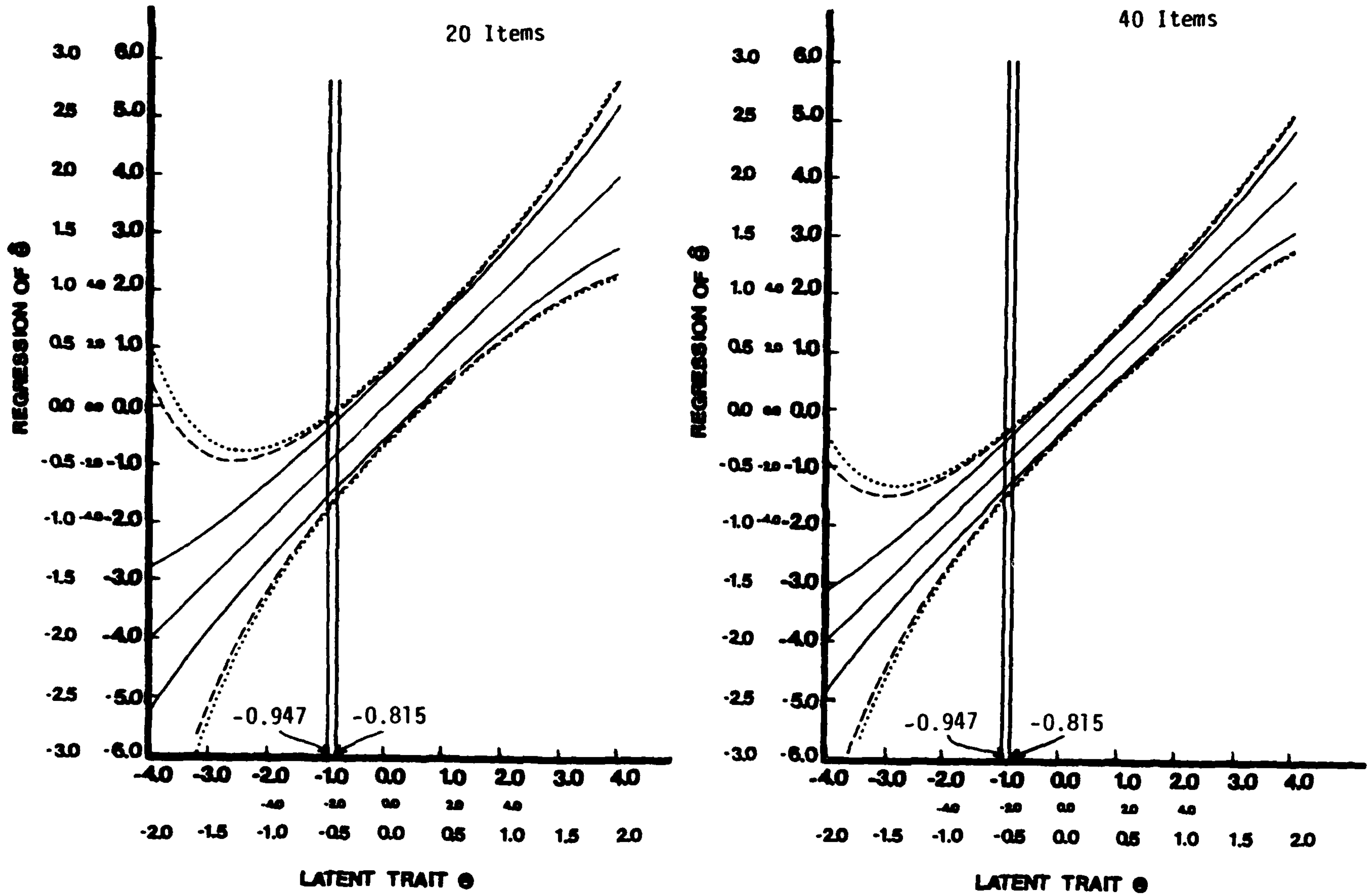


FIGURE 3-2

Standard Errors of Estimation Taken on Both Upper and Lower Sides of the "Unbiased" Regression of $\hat{\theta}$ on θ, in the Normal Ogive Model (Solid Line), in the Three-Parameter Logistic Model with $c_g = 0.20$ (Dashed Line) and with $c_g = 0.25$ (Dotted Line), for Each of the Ten Tests of Equivalent Items. The Two Values of θ_g Are Also Shown.

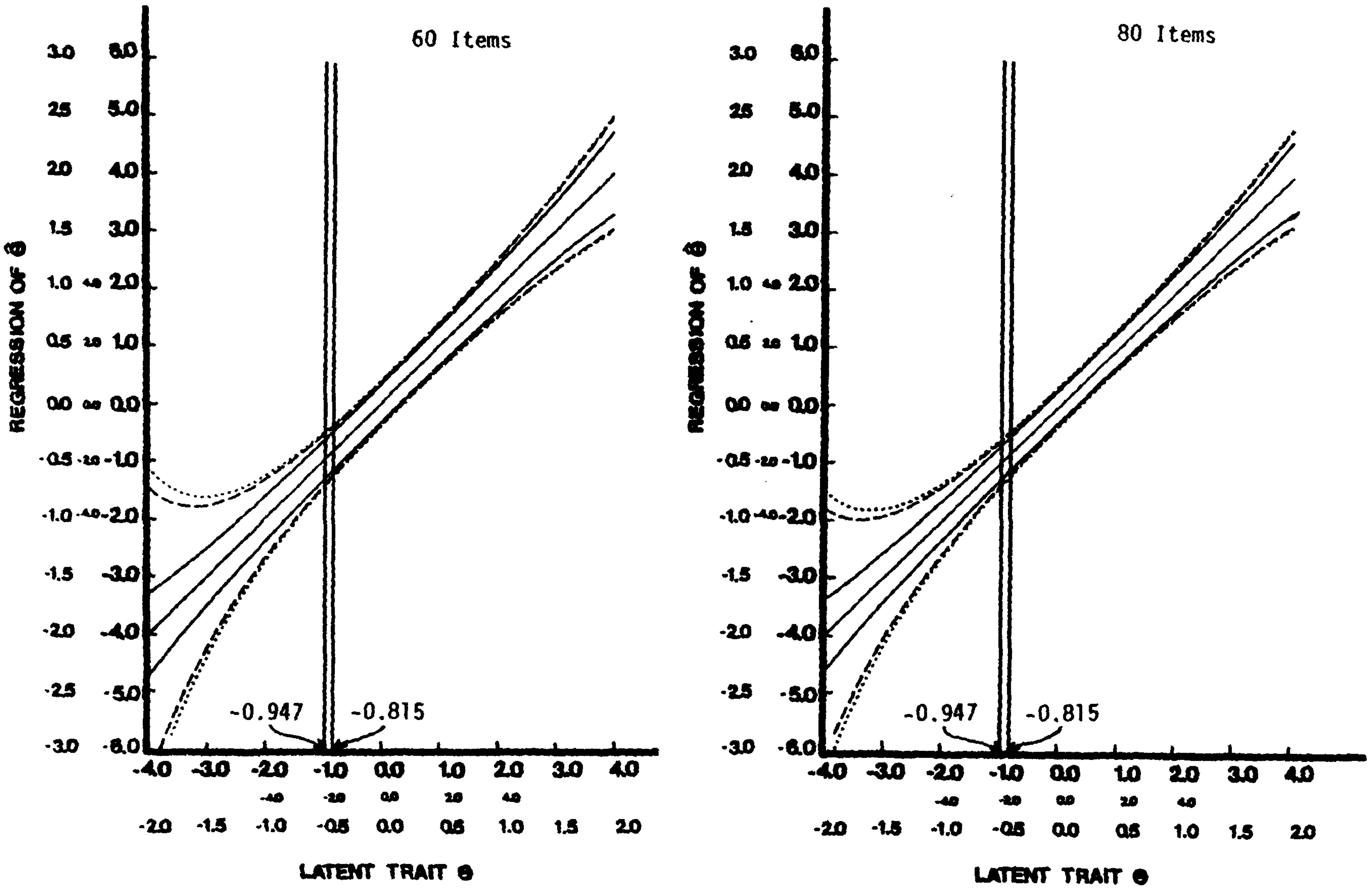


FIGURE 3-2 (Continued)

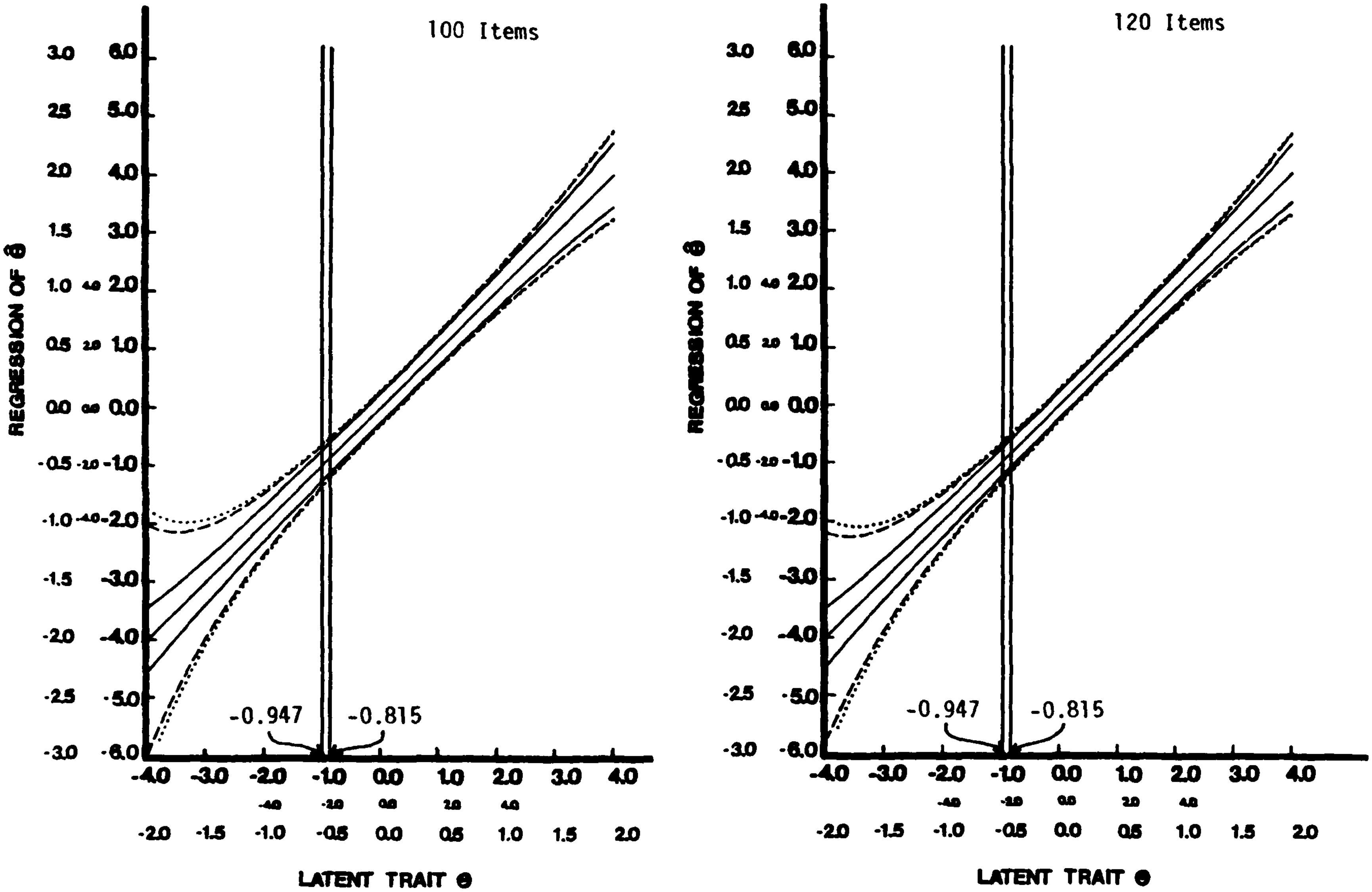


FIGURE 3-2 (Continued)

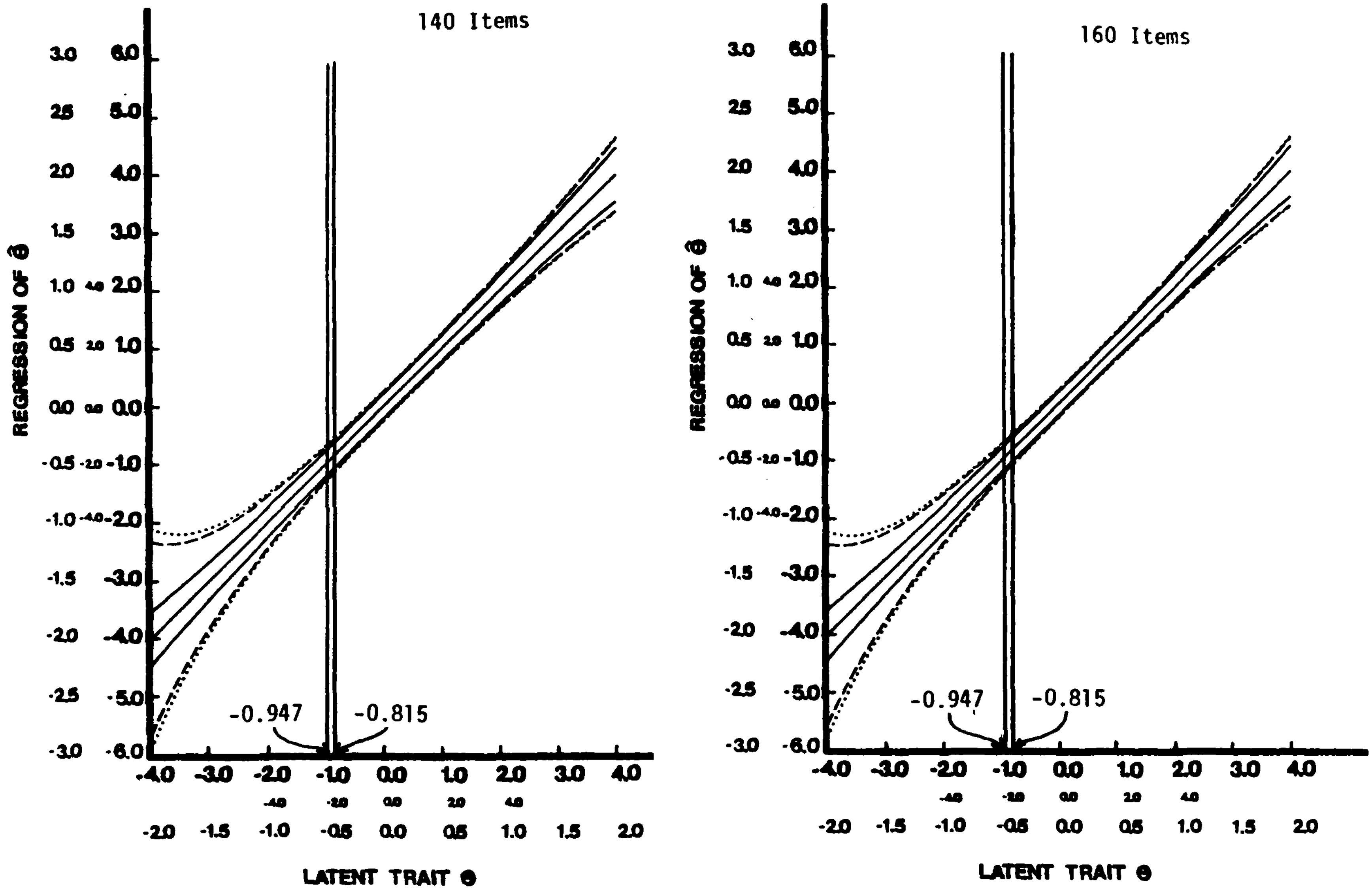
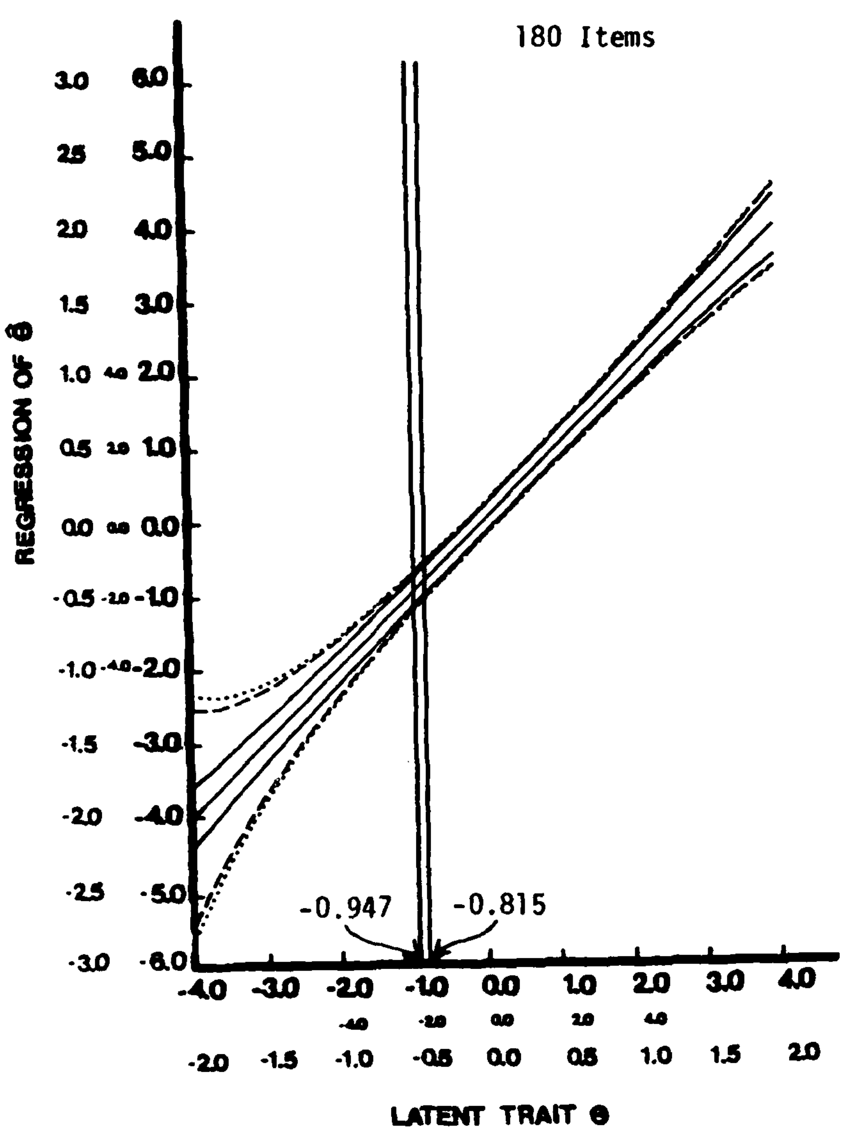


FIGURE 3-2 (Continued)



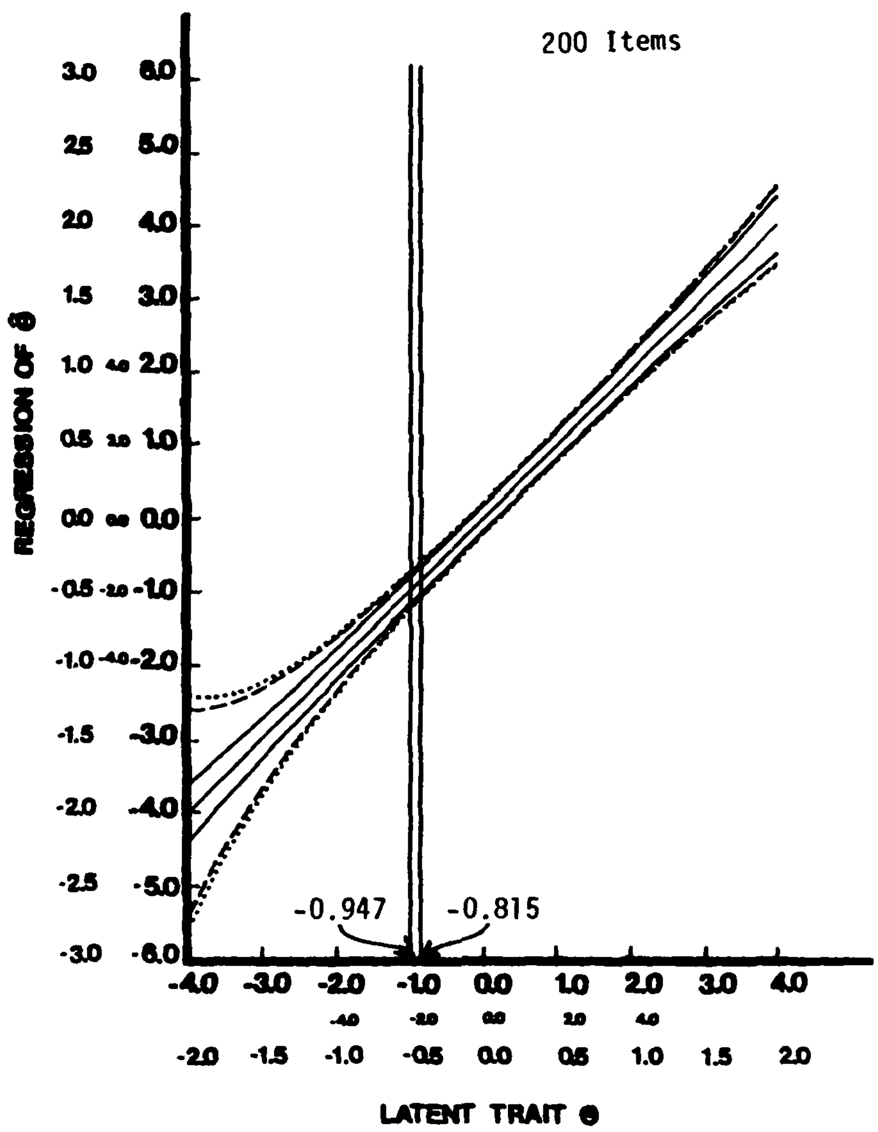


FIGURE 3-2 (Continued)

dashed and dotted lines for the three-parameter logistic model with $c_g = 0.20$ and $c_g = 0.25$, respectively, for each of the ten tests of equivalent items. The number of items in those ten tests are 20, 40, 60, 80, 100, 120, 140, 160, 180 and 200, respectively. The common item parameters are the same as those used in the eleven tests in Figure 3-1, i.e., $a_g = 0.50$ and $b_g = 0.00$. On both the abscissa and the ordinate of Figure 3-2, two additional sets of numbers are shown in the smallest and medium sizes to adjust the scales to the change of a_g from 0.50 to 2.00 and 1.00, respectively.

We can see in these ten graphs of Figure 3-2 that for relatively large numbers of items the three-parameter logistic model provides us with reasonably small confidence intervals for certain intervals of θ, but the accuracy of estimation drops radically as θ departs from θ_g in the negative direction. Although this tendency is conspicuous for smaller numbers of items, it is still clear for larger numbers of items, like 200. The effect of noise caused by random guessing is obvious, and we need to devise some way to make up for it if we must use the three-parameter logistic model.

IV <u>Loss in Speed of Convergence of the Conditional Distribution of the Maximum Likelihood Estimate to the Normality</u>

In the preceding section, we have observed the loss of accuracy in ability estimation caused by random guessing, which is embedded in the three-parameter logistic model. In so doing, we compared the normal ogive model and the three-parameter logistic model with $c_g = 0.20$ and $c_g = 0.25$, with respect to the confidence interval of the maximum

likelihood estimate $\hat{\theta}_t$, given θ , which was approximated by the unbiased regression and the reciprocal of the square root of the test information function as the standard error of estimation. Those confidence intervals are approximations, using the asymptotic normality of the maximum likelihood estimate, given θ .

In this section, we shall focus our attention on the loss in the speed of convergence of the conditional distribution of the maximum likelihood estimate $\hat{\theta}_t$ to the normality, which is caused by random guessing embedded in the three-parameter logistic model. In so doing, we shall use five hypothetical tests of equivalent items, i.e., $n = 20, 40, 80, 120, 200$, which were chosen out of the ten tests used in the preceding section.

We notice that, with equivalent items, the conditional distribution of the test score t , given θ , is binomial, with n and $P_g(\theta)$ as the two parameters. Since there is one-to-one correspondence between the test score t and the maximum likelihood estimate $\hat{\theta}_t$, the probability function of t also applies to the probability function of $\hat{\theta}_t$, if each non-zero probability is assigned to $\hat{\theta}_t$ instead of t . Questions are raised as to how close this discrete distribution is to $N(\theta, \{I(\theta)\}^{-1/2})$, and if there are substantial differences between the two models with respect to the speed of convergence to the normality.

We notice that in the normal ogive model $\hat{\theta}_t$ assumes negative and positive infinities for $t = 0$ and $t = n$, respectively, whereas in the three-parameter logistic model it does for $t < nc_g$ and $t = n$. One measure of the speed of convergence to the normality, therefore, is the

probabilities assigned to the negative and positive infinities in each distribution, i.e., the greater these probabilties are, the slower the convergence to the normality is. Table 4-1 presents these probabilities assigned to the negative and positive infinities at each of the sixteen equally spaced points of θ for each of the five hypothetical tests of equivalent items. We can see from this table that, in all five situations, where the numbers of items are 20, 40, 80, 120 and 200 , respectively, the sum totals of the probabilities assigned to the negative or positive infinity on the three-parameter logistic model are substantially larger, compared with the ones on the normal ogive model.

Since the probabilities assigned to the negative and positive infinities are not zero in each conditional distribution of $\hat{\theta}_t$, given θ , the moments of each distribution are indeterminate. As a crude measure, however, we shall compute the moments of each distribution by simply "ignoring" the negative and positive infinities and their corresponding probabilities. Tables 4-2 through 4-4 present the first moment about the origin and the second through fourth moments about the mean thus computed, for each of the sixteen values of θ . For simplicity, the symbols μ_1' , μ_2 , μ_3 and μ_4 are used for those moments, which indicate

$$(4.1) \qquad \mu_1' = E(\hat{\theta}_t \mid \theta) \quad ,$$

and

TABLE 4-1

Sum Totals of Probabilities Assigned to Negative and Positive Infinities and Their Sum in the Conditional Distribution of $\hat{\theta}_t$, given θ , at Sixteen Different Values of θ , on the Normal Ogive Model and on the Three-Parameter Logistic Model with $c_g = 0.20$ and $c_g = 0.25$. In all Three Cases, Items Are Equivalent with $a_g = 0.50$ and $b_g = 0.00$. 20 Items.

Negative Infinity			Positive Infinity			Total		
Normal Ogive	3-P.L. c_g= 0.20	3-P.L. c_g= 0.25	Normal Ogive	3-P.L. c_g= 0.20	3-P.L. c_g= 0.25	Normal Ogive	3-P.L. c_g= 0.20	3-P.L. c_g= 0.25
0.250858	0.383241	0.399937	0.000000	0.000000	0.000000	0.250858	0.383241	0.399937
0.130519	0.305054	0.328158	0.000000	0.000000	0.000000	0.130519	0.305054	0.328158
0.054154	0.217702	0.244937	0.000000	0.000000	0.000000	0.054154	0.217702	0.244937
0.017107	0.132637	0.159213	0.000000	0.000000	0.000000	0.017107	0.132637	0.159213
0.003925	0.064763	0.085083	0.000000	0.000000	0.000000	0.003925	0.064763	0.085083
0.000624	0.023528	0.034868	0.000000	0.000000	0.000000	0.000624	0.023528	0.034869
0.000066	0.005886	0.010171	0.000000	0.000001	0.000003	0.000066	0.005887	0.010175
0.000004	0.000947	0.001973	0.000000	0.000011	0.000029	0.000005	0.000959	0.002002
0.000000	0.000094	0.000242	0.000004	0.000110	0.000223	0.000005	0.000203	0.000466
0.000000	0.000006	0.000019	0.000066	0.000794	0.001348	0.000066	0.000800	0.001367
0.000000	0.000000	0.000001	0.000624	0.004183	0.006177	0.000624	0.004183	0.006178
0.000000	0.000000	0.000000	0.003925	0.016070	0.021363	0.003925	0.016070	0.021363
0.000000	0.000000	0.000C00	0.017107	0.046318	0.056940	0.017107	0.046318	0.056940
0.000000	0.000000	0.000000	0.054154	0.104388	0.121155	0.054154	0.104388	0.121155
0.000000	0.000000	0.000000	0.130519	0.192483	0.214233	0.130519	0.192483	0.214233
0.000000	0.000000	0.000000	0.250858	0.303082	0.327254	0.250858	0.303082	0.327254

Table 4-1 (Continued) 40 Items.

Negative Infinity			Positive Infinity			Total		
Normal Ogive	3-P.L. c_g= 0.20	3-P.L. c_g= 0.25	Normal Ogive	3-P.L. c_g= 0.20	3-P.L. c_g= 0.25	Normal Ogive	3-P.L. c_g= 0.20	3-P.L. c_g= 0.25
0.062930	0.261408	0.288228	0.000000	0.000000	0.000000	0.062930	0.261408	0.288228
0.017035	0.174648	0.204328	0.000000	0.000000	0.000000	0.017035	0.174648	0.204328
0.002933	0.094627	0.121081	0.000000	0.000000	0.000000	0.002933	0.094627	0.121081
0.000293	0.037602	0.054863	0.000000	0.000000	0.000000	0.000293	0.037602	0.054863
0.000015	0.009617	0.016877	0.000000	0.000000	0.000000	0.000015	0.009617	0.016877
0.000000	0.001358	0.003052	0.000000	0.000000	0.000000	0.000000	0.001358	0.003052
0.000000	0.000090	0.000278	0.000000	0.000000	0.000000	0.000000	0.000090	0.000278
0.000000	0.000002	0.000011	0.000000	0.000000	0.000000	0.000000	0.000002	0.000011
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000001	0.000002	0.000000	0.000001	0.000002
0.000000	0.000000	0.000000	0.000000	0.000017	0.000038	0.000000	0.000017	0.000038
0.000000	0.000000	0.000000	0.000015	0.000258	0.000456	0.000015	0.000258	0.000456
0.000000	0.000000	0.000000	0.000293	0.002145	0.003242	0.000293	0.002145	0.003242
0.000000	0.000000	0.000000	0.002933	0.010897	0.014679	0.002933	0.010897	0.014679
0.000000	0.000000	0.000000	0.017035	0.037050	0.045896	0.017035	0.037050	0.045896
0.000000	0.000000	0.000000	0.062930	0.091859	0.107095	0.062930	0.091859	0.107095

Table 4-1 (Continued) 80 Items.

Negative Infinity			Positive Infinity			Total		
Normal Ogive	3-P.L. c_g= 0.20	3-P.L. c_g= 0.25	Normal Ogive	3-P.L. c_g= 0.20	3-P.L. c_g= 0.25	Normal Ogive	3-P.L. c_g= 0.20	3-P.L. c_g= 0.25
0.003960	0.144555	0.175542	0.000000	0.000000	0.000000	0.003960	0.144555	0.175542
0.000290	0.069838	0.095298	0.000000	0.000000	0.000000	0.000290	0.069838	0.095298
0.000009	0.022414	0.036598	0.000000	0.000000	0.000000	0.000009	0.022414	0.036598
0.000000	0.003888	0.008284	0.000000	0.000000	0.000000	0.000000	0.003888	0.008284
0.000000	0.000279	0.000866	0.000000	0.000000	0.000000	0.000000	0.000279	0.000866
0.000000	0.000006	0.000031	0.000000	0.000000	0.000000	0.000000	0.000006	0.000031
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000005	0.000011	0.000000	0.000005	0.000011
0.000000	0.000000	0.000000	0.000009	0.000119	0.000215	0.000009	0.000119	0.000215
0.000000	0.000000	0.000000	0.000290	0.001373	0.002106	0.000290	0.001373	0.002106
0.000000	0.000000	0.000000	0.003960	0.008438	0.011469	0.003960	0.008438	0.011469

Table 4-1 (Continued) 120 Items.

Negative Infinity			Positive Infinity			Total		
Normal Ogive	3-P.L. $c_g = 0.20$	3-P.L. $c_g = 0.25$	Normal Ogive	3-P.L. $c_g = 0.20$	3-P.L. $c_g = 0.25$	Normal Ogive	3-P.L. $c_g = 0.20$	3-P.L. $c_g = 0.25$
0.000249	0.086530	0.114982	0.000000	0.000000	0.000000	0.000249	0.086530	0.114982
0.000005	0.030616	0.048413	0.000000	0.000000	0.000000	0.000005	0.030616	0.048413
0.000000	0.005892	0.012206	0.000000	0.000000	0.000000	0.000000	0.005892	0.012206
0.000000	0.000451	0.001396	0.000000	0.000000	0.000000	0.000000	0.000451	0.001396
0.000000	0.000009	0.000050	0.000000	0.000000	0.000000	0.000000	0.000009	0.000050
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000001	0.000003	0.000000	0.000001	0.000003
0.000000	0.000000	0.000000	0.000005	0.000051	0.000097	0.000005	0.000051	0.000097
0.000000	0.000000	0.000000	0.000249	0.000775	0.001228	0.000249	0.000775	0.001228

Table 4-1 (Continued) 200 Items.

Negative Infinity			Positive Infinity			Total		
Normal Ogive	3-P.L. $c_g = 0.20$	3-P.L. $c_g = 0.25$	Normal Ogive	3-P.L. $c_g = 0.20$	3-P.L. $c_g = 0.25$	Normal Ogive	3-P.L. $c_g = 0.20$	3-P.L. $c_g = 0.25$
0.000001	0.033982	0.053680	0.000000	0.000000	0.000000	0.000001	0.033982	0.053680
0.000000	0.006540	0.013795	0.000000	0.000000	0.000000	0.000000	0.006540	0.013795
0.000000	0.000458	0.001519	0.000000	0.000000	0.000000	0.000000	0.000458	0.001519
0.000000	0.000007	0.000045	0.000000	0.000000	0.000000	0.000000	0.000007	0.000045
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000001	0.000007	0.000014	0.000001	0.000007	0.000014

Table 4-2

Conditional Mean μ_1', Second to Fourth Moments, μ_2, μ_3 and μ_4, Indices β_1 and β_2, Pearson's Criterion κ and Pearson's Type of Distribution Obtained at Sixteen Different Values of θ, for Each of the Five Hypothetical Tests of Equivalent Items on the Normal Ogive Model with $a_g = 0.50$ and $b_g = 0.00$. Moments Were Calculated by "Ignoring" Negative and Positive Infinities.

The Mark ** Indicates that the Sum Total of the Probabilities for Finite Values of $\hat{\theta}_t$ Is Greater Than, or Equal to, 0.999999, and * Means It Is Greater Than, or Equal to, 0.99 and Less Than 0.999999. 20 Items.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type	
[illegible]	[illegible]	[illegible] D 01	[illegible]	-0.6203573669D 00	0.7608027757D 00	0.2062539602D 01	0.2242789183D 01	-0.6610954323D 00	1	
[illegible]	[illegible]	[illegible] D 01	[illegible]	-0.3041226249D 00	0.4202826046D 00	0.1255977797D 01	0.2392201450D 01	-0.3158064568D 00	1	
[illegible]	[illegible]	[illegible] D 01	[illegible]	-0.1641589340D 00	0.4607093803D 00	0.3553937152D 00	0.2571663768D 01	-0.1555679349D 00	1	
[illegible]	[illegible]	[illegible] D 01	[illegible]	-0.1256229268D 00	0.5718112753D 00	0.1795968380D 00	0.2893076803D 01	-0.1876879930D 00	1	
[illegible]	[illegible]	[illegible]	[illegible]	-0.1162318753D 00	0.6011208391D 00	0.1674558438D 00	0.3291117461D 01	0.1638588943D 01	6	*
[illegible]	[illegible]	[illegible]	[illegible]	-0.8430451624D-01	0.5327171776D 00	0.1190519856D 00	0.3495450289D 01	0.1458888881D 00	4	*
[illegible]	[illegible]	[illegible]	[illegible]	[illegible] D-01	0.4454386185D 00	0.4595879573D-01	0.3435663452D 01	0.4755916383D-01	4	*
[illegible]	[illegible]	[illegible]	[illegible]	-0.14[illegible]D-01	0.3972025680D 00	0.4930025122D-02	0.3335786280D 01	0.5662396201D-02	4	*
[illegible]	[illegible]	[illegible]	[illegible]	0.1424360157D-01	0.3972075680D 00	0.4930025122D-02	0.3335788268D 01	0.5662396210D-02	4	*
[illegible]	[illegible]	[illegible]	[illegible]	0.1536752219D-01	0.4454386184D 00	0.4595875373D-01	0.3435663452D 01	0.4755916382D-01	4	*
[illegible]	[illegible]	[illegible]	[illegible]	0.8430451624D-01	0.5327171776D 00	0.1190519856D 00	0.3495450268D 01	[illegible]5088881D 00	4	*
[illegible]	[illegible]	[illegible]	[illegible]	0.1162318753D 00	0.6011208391D 00	0.1674558438D 00	0.3291117461D 01	0.1638588943D 01	6	*
[illegible]	[illegible]	0.18[illegible]26001D [illegible]	[illegible]	0.1256229268D 00	0.5718112753D 00	0.1795968380D 00	0.2893076803D 01	-0.1877687993D 00	1	
[illegible]	[illegible]	0.212[illegible]66880 [illegible]	[illegible]592265D 00	0.1641589340D 00	0.4607093803D 00	0.3553937152D 00	0.2571663768D 01	-0.1555679349D 00	1	
[illegible]	[illegible]	0.2718547376D 01	[illegible]5228000D 00	0.3041226249D 00	0.4202826046D 00	0.1255977797D 01	0.2392201450D 01	-0.3158084568D 00	1	
[illegible]	[illegible]	0.2090324687D 01	0.5824278266D 00	0.6393573669D 00	0.7608027757D 00	0.2062539602D 01	0.2242789183D 01	-0.6610954323D 00	1	

Table 4-2 (Continued) 40 Items.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type	
1	-3.0	-0.2852838963D 01	0.3271584280D 00	-0.1887173500D 00	0.2840697110D 00	0.1017066432D 01	0.2654047577D 01	-0.2870568533D 00	1	
2	-2.6	-0.2634615130D 01	0.3176306772D 00	-0.1124852092D 00	0.3037624759D 00	0.3948421600D 00	0.3010850855D 01	-0.2824465317D 00	1	
3	-2.2	-0.2266076149D 01	0.2862295758D 00	-0.8730613238D-01	0.2886416935D 00	0.3250468627D 00	0.3523141209D 01	0.3705322000D 01	6	*
4	-1.8	-0.1852545573D 01	0.2409191797D 00	-0.5762116916D-01	0.2149897721D 00	0.2374378795D 00	0.3704034711D 01	0.2718746904D 00	4	*
5	-1.4	-0.1435964279D 01	0.2056025529D 00	-0.3231600296D-01	0.1477172646D 00	0.1201571917D 00	0.3494413151D 01	0.1480735567D 00	4	*
6	-1.0	-0.1023107961D 01	0.1832770572D 00	-0.1747668491D-01	0.1102766899D 00	0.4961280751D-01	0.3282974546D 01	0.9041407778D-01	4	**
7	-0.6	-0.6129407515D 00	0.1703786602D 00	-0.8781663686D-02	0.9216792240D-01	0.1559222184D-01	0.3175041457D 01	0.3872929098D-01	4	**
8	-0.2	-0.2041702951D 00	0.1644413659D 00	-0.2689577344D-02	0.8469061268D-01	0.1626802567D-02	0.3131937765D 01	0.4714925698D-02	4	**
9	0.2	0.2041702951D 00	0.1644413659D 00	0.2689577344D-02	0.8469061268D-01	0.1626802567D-02	0.3131937765D 01	0.4714925698D-02	4	**
10	0.6	0.6129407515D 00	0.1703786602D 00	0.8781663686D-02	0.9216792240D-01	0.1559222184D-01	0.3175041457D 01	0.3872929098D-01	4	**
11	1.0	0.1023107961D 01	0.1832770572D 00	0.1747668491D-01	0.1102766899D 00	0.4961280751D-01	0.3282974546D 01	0.9041407778D-01	4	**
12	1.4	0.1435964279D 01	0.2056025529D 00	0.3231600296D-01	0.1477172646D 00	0.1201571917D 00	0.3494413151D 01	0.1480735567D 00	4	*
13	1.8	0.1852545573D 01	0.2409191797D 00	0.5762116916D-01	0.2149897721D 00	0.2374378795D 00	0.3704034711D 01	0.2718746904D 00	4	*
14	2.2	0.2266076149D 01	0.2862295758D 00	0.8730613238D-01	0.2886416935D 00	0.3250468627D 00	0.3523141209D 01	0.3705322000D 01	6	*
15	2.6	0.2634615130D 01	0.3176306772D 00	0.1124852092D 00	0.3037624759D 00	0.3948421600D 00	0.3010850855D 01	-0.2824465317D 00	1	
16	3.0	0.2852838963D 01	0.3271584280D 00	0.1887173500D 00	0.2840697110D 00	0.1017066432D 01	0.2654047577D 01	-0.2870568533D 00	1	

Table 4-2 (Continued) 80 Items.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type	
1	-3.0	-0.3061450503D 01	0.2182982052D 00	-0.6995074161D-01	0.1717875998D 00	0.4703640826D 00	0.3604885556D 01	-0.1958798807D 01	1	*
2	-2.6	-0.2652758925D 01	0.1715539677D 00	-0.4200016846D-01	0.1147030196D 00	0.3493813866D 00	0.3897385072D 01	0.3827361752D 00	4	*
3	-2.2	-0.2236642725D 01	0.1354812863D 00	-0.2148127986D-01	0.6607783882D-01	0.1855591492D 00	0.3599956124D 01	0.2269342845D 00	4	*
4	-1.8	-0.1825203674D 01	0.1125770063D 00	-0.1140563883D-01	0.4209580870D-01	0.9117801721D-01	0.3321539842D 01	0.1894274502D 00	4	**
5	-1.4	-0.1417158971D 01	0.9796594092D-01	-0.6463772790D-02	0.3055307541D-01	0.4443721389D-01	0.3183498898D 01	0.1442541465D 00	4	**
6	-1.0	-0.1011126440D 01	0.8864387669D-01	-0.3694631109D-02	0.2445982099D-01	0.1959728654D-01	0.3112832789D 01	0.8852638666D-01	4	**
7	-0.6	-0.6062673162D 00	0.8307175843D-01	-0.1921164208D-02	0.2122672619D-01	0.6438263741D-02	0.3075928320D 01	0.3649447603D-01	4	**
8	-0.2	-0.2020249305D 00	0.8045691464D-01	-0.5970121964D-03	0.1980747426D-01	0.6843468684D-03	0.3059865604D 01	0.4362715499D-02	4	**
9	0.2	0.2020249305D 00	0.8045691464D-01	0.5970121964D-03	0.1980747426D-01	0.6843468684D-03	0.3059865604D 01	0.4362715499D-02	4	**
10	0.6	0.6062673162D 00	0.8307175843D-01	0.1921164208D-02	0.2122672619D-01	0.6438263741D-02	0.3075928320D 01	0.3649447603D-01	4	**
11	1.0	0.1011126440D 01	0.8864387669D-01	0.3694631109D-02	0.2445982099D-01	0.1959728654D-01	0.3112832789D 01	0.8852638666D-01	4	**
12	1.4	0.1417158971D 01	0.9796594092D-01	0.6463772790D-02	0.3055307541D-01	0.4443721389D-01	0.3183498898D 01	0.1442541465D 00	4	**
13	1.8	0.1825203674D 01	0.1125770063D 00	0.1140563883D-01	0.4209580870D-01	0.9117801721D-01	0.3321539842D 01	0.1894274502D 00	4	**
14	2.2	0.2236642725D 01	0.1354812863D 00	0.2148127986D-01	0.6607783882D-01	0.1855591492D 00	0.3599956124D 01	0.2269342845D 00	4	*
15	2.6	0.2652758925D 01	0.1715539677D 00	0.4200016846D-01	0.1147030196D 00	0.3493813866D 00	0.3897385072D 01	0.3827361752D 00	4	*
16	3.0	0.3061450503D 01	0.2182982052D 00	0.6995074161D-01	0.1717875998D 00	0.4703640826D 00	0.3604885556D 01	-0.1958798807D 01	1	*

Table 4-2 (Continued) 120 Items.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type	
1	-3.0	-0.3050562378D 01	0.1435106079D 00	-0.3408383283D-01	0.8211366361D-01	0.3930473212D 00	0.3987010881D 01	0.4086185073D 00	4	*
2	-2.6	-0.2634604929D 01	0.1088181522D 00	-0.1623621173D-01	0.4291532781D-01	0.2045810348D 00	0.3624179840D 01	0.2547273730D 00	4	*
3	-2.2	-0.2223774113D 01	0.8726259415D-01	-0.8279425830D-02	0.2532818425D-01	0.1031611277D 00	0.3326195852D 01	0.2316205143D 00	4	**
4	-1.8	-0.1816478924D 01	0.7345731965D-01	-0.4617825393D-02	0.1718679977D-01	0.5379845239D-01	0.3185113131D 01	0.1958674307D 00	4	**
5	-1.4	-0.1411274629D 01	0.6438514242D-01	-0.2693648708D-02	0.1289928152D-01	0.2718472419D-01	0.3111674779D 01	0.1447851771D 00	4	**
6	-1.0	-0.1007331922D 01	0.5849616566D-01	-0.1563798199D-02	0.1050780937D-01	0.1221741198D-01	0.3070841395D 01	0.8751485738D-01	4	**
7	-0.6	-0.6041366202D 00	0.5494234701D-01	-0.8199997566D-03	0.9202770070D-02	0.4054206114D-02	0.3048626050D 01	0.3577277716D-01	4	**
8	-0.2	-0.2013374948D 00	0.5326644819D-01	-0.2557681346D-03	0.8621939094D-02	0.4328442526D-03	0.3038767498D 01	0.4258870876D-02	4	**
9	0.2	0.2013374948D 00	0.5326644819D-01	0.2557681346D-03	0.8621939094D-02	0.4328442526D-03	0.3038767498D 01	0.4258870876D-02	4	**
10	0.6	0.6041366202D 00	0.5494234701D-01	0.8199997566D-03	0.9202770070D-02	0.4054206114D-02	0.3048626050D 01	0.3577277716D-01	4	**
11	1.0	0.1007331922D 01	0.5849616566D-01	0.1563798199D-02	0.1050780937D-01	0.1221741198D-01	0.3070841395D 01	0.8751485738D-01	4	**
12	1.4	0.1411274629D 01	0.6438514242D-01	0.2693648708D-02	0.1289928152D-01	0.2718472419D-01	0.3111674779D 01	0.1447851771D 00	4	**
13	1.8	0.1816478924D 01	0.7345731965D-01	0.4617825393D-02	0.1718679977D-01	0.5379845239D-01	0.3185113131D 01	0.1958674307D 00	4	**
14	2.2	0.2223774113D 01	0.8726259415D-01	0.8279425830D-02	0.2532818425D-01	0.1031611277D 00	0.3326195852D 01	0.2316205143D 00	4	**
15	2.6	0.2634604929D 01	0.1088181522D 00	0.1623621173D-01	0.4291532781D-01	0.2045810348D 00	0.3624179840D 01	0.2547273730D 00	4	*
16	3.0	0.3050562378D 01	0.1435106079D 00	0.3408383283D-01	0.8211366361D-01	0.3930473212D 00	0.3987010881D 01	0.4086185073D 00	4	*

Table 4-2 (Continued) 200 Items.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type	
1	-3.0	-0.3029621652D 01	0.8082532136D-01	-0.1013144854D-01	0.2329419244D-01	0.1944020166D 00	0.3565765502D 01	0.2793135304D 00	4	**
2	-2.6	-0.2620144614D 01	0.6268792522D-01	-0.4973294016D-02	0.1294095724D-01	0.1004006632D 00	0.3293052160D 01	0.2710757929D 00	4	**
3	-2.2	-0.2213975491D 01	0.5108210525D-01	-0.2697948308D-02	0.8265348074D-02	0.5460858031D-01	0.3167550681D 01	0.2424372126D 00	4	**
4	-1.8	-0.1809744103D 01	0.4337804404D-01	-0.1555778721D-02	0.5834803632D-02	0.2965416900D-01	0.3100889668D 01	0.1986177437D 00	4	**
5	-1.4	-0.140663[illegible] 01	0.3821302621D-01	-0.9243873760D-03	0.4472458209D-02	0.1531347363D-01	0.3062833769D 01	0.1446129416D 00	4	**
6	-1.0	-0.1004359687D 01	0.3482131670D-01	-0.5424124976D-03	0.3686885390D-02	0.5968251350D-02	0.3040665790D 01	0.8662925438D-01	4	**
7	-0.6	-0.60[illegible]5266[illegible]D 00	0.3276071[illegible]57D-01	-0.2861266016D-03	0.3250172674D-02	0.2328394038D-02	0.3028305234D 01	0.3521069409D-01	4	**
8	[illegible]	-0.2007966923D 00	0.3178554078D-01	-0.8948761640D-04	0.3053944236D-02	0.2493659011D-03	0.3022747658D 01	0.1179894744D-02	4	**
9	[illegible]	0.2007966922D 00	0.3178554078D-01	0.8948761640D-04	0.3053944236D-02	0.2493659011D-03	0.3022747658D 01	0.1179894744D-02	4	**
10	[illegible]	0.6024526654[illegible]	[illegible]27607166[illegible]	0.2861266016D-03	0.3250172674D-02	0.2328393[illegible]D-02	0.3028305234D 01	0.3521069409D-01	4	**
11	[illegible]	0.[illegible]004359687[illegible]	0.348213167[illegible]D-01	0.3424124976D-03	0.3686885390D-02	0.6968251350D-02	0.3040669790D 01	0.8662925438D-01	4	**
12	1.4	0.140663[illegible]	0.3821302621D-01	0.9243873760D-03	0.4472458209D-02	0.1531347363D-01	0.3062833769D 01	0.1446129416D 00	4	**
13	1.8	0.1809744[illegible]	0.4337804404D-01	0.1555778721D-02	0.5834803632D-02	0.2965416900D-01	0.3100889668D 01	0.1986177437D 00	4	**
14	2.2	0.22139754[illegible] 01	0.5108210525D-01	0.2697948308D-02	0.8265348074D-02	0.5460858031D-01	0.3167550681D 01	0.2424372125D 00	4	**
15	2.6	0.2620144615D 01	0.6268792522D-01	0.4973294016D-02	0.1294095724D-01	0.1004006632D 00	0.3293052160D 01	0.2710757929D 00	4	**
16	3.0	0.3029621652D 01	0.8082532136D-01	0.1013144854D-01	0.2329419244D-01	0.1944020166D 00	0.3565765502D 01	0.2793135304D 00	4	**

Table 4-3

Conditional Mean μ_1', Second to Fourth Moments, μ_2, μ_3 and μ_4, Indices β_1 and β_2, Pearson's Criterion κ and Pearson's Type of Distribution Obtained at Sixteen Different Values of θ, for Each of the Five Hypothetical Tests of Equivalent Items on the Three-Parameter Logistic Model with $a_g = 0.50$, $b_g = 0.00$ and $c_g = 0.20$. Moments Were Calculated by "Ignoring" Negative and Positive Infinities. The Mark ** Indicates That the Sum Total of the Probabilities for Finite Values of $\hat{\theta}_t$ Is Greater Than, or Equal to, 0.999999, and * Means It Is Greater Than, or Equal to, 0.99 and Less Than 0.999999. 20 Items.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type	
1	-3.0	-0.1380745760D 01	0.8380039935D 00	-0.1304965575D 01	0.2279942156D 01	0.2893742252D 01	0.3246621439D 01	-0.8007813812D 00	1	
2	-2.6	-0.1473213007D 01	0.7512783492D 00	-0.1034613873D 01	0.1797881015D 01	0.2524375606D 01	0.3185364971D 01	-0.6486301059D 00	1	
3	-2.2	-0.1528935763D 01	0.6926577913D 00	-0.7717606421D 00	0.1482905472D 01	0.1792293305D 01	0.3090836415D 01	-0.4579758368D 00	1	
4	-1.8	-0.1499269840D 01	0.6831650135D 00	-0.5505663896D 00	0.1404816156D 01	0.9506979122D 00	0.3010012224D 01	-0.3299214239D 00	1	
5	-1.4	-0.1339246182D 01	0.6954196497D 00	-0.4061337345D 00	0.1493023963D 01	0.4904528421D 00	0.3087257559D 01	-0.3220750870D 00	1	
6	-1.0	-0.1041668878D 01	0.6717125818D 00	-0.3298955964D 00	0.1541719644D 01	0.3590894017D 00	0.3416948553D 01	-0.1206387508D 01	1	
7	-0.6	-0.6492371601D 00	0.5990642482D 00	-0.2432142239D 00	0.1380994156D 01	0.2751425352D 00	0.3848088370D 01	0.2543170596D 00	4	*
8	-0.2	-0.2193070231D 00	0.5229979092D 00	-0.1169524853D 00	0.1090982988D 01	0.9561346391D-01	0.3988577851D 01	0.4408288923D-01	4	*
9	0.2	0.2160874234D 00	0.4865839731D 00	0.6306793983D-02	0.9170442541D 00	0.3452581098D-03	0.3873242545D 01	0.1507975706D-03	4	*
10	0.6	0.6498799163D 00	0.4978425395D 00	0.1089189904D 00	0.9391241098D 00	0.9614599631D-01	0.3789125446D 01	0.5777204579D-01	4	*
11	1.0	0.1077110256D 01	0.5392247251D 00	0.1835643024D 00	0.1026344758D 01	0.2149151657D 00	0.3529829050D 01	0.4097690286D 00	4	*
12	1.4	0.1474516962D 01	0.5739333389D 00	0.2125046875D 00	0.1006011192D 01	0.2388650830D 00	0.3054076466D 01	-0.3128111028D 00	1	
13	1.8	0.1792874799D 01	0.5664326230D 00	0.2311341778D 00	0.8490758061D 00	0.2939568362D 00	0.2646366255D 01	-0.1519383520D 00	1	
14	2.2	0.1975005766D 01	0.5316820861D 00	0.3209396118D 00	0.6867856671D 00	0.6853155121D 00	0.2429501476D 01	-0.2061911385D 00	1	
15	2.6	0.1990866714D 01	0.5540806165D 00	0.5235744219D 00	0.7238524079D 00	0.1611528430D 01	0.2357784501D 01	-0.4111824870D 00	1	
16	3.0	0.1856559944D 01	0.7125309430D 00	0.8846593717D 00	0.1179414488D 01	0.2163420685D 01	0.2323052368D 01	-0.6972623065D 00	1	

Table 4-3 (Continued) 40 Items.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type	
1	-3.0	-0.1979041597D 01	0.8863064013D 00	-0.1437727915D 01	0.2761481570D 01	0.2968942265D 01	0.3515396994D 01	-0.7760817329D 00	1	
2	-2.6	-0.2054962301D 01	0.7502677653D 00	-0.1015719681D 01	0.2023146265D 01	0.2442861654D 01	0.3594137652D 01	-0.6136218938D 00	1	
3	-2.2	-0.2019189471D 01	0.6753918450D 00	-0.6950172763D 00	0.1648842003D 01	0.1567919506D 01	0.3614659701D 01	-0.5060251876D 00	1	
4	-1.8	-0.1821885421D 01	0.6121317327D 00	-0.4883289042D 00	0.1446543281D 01	0.1039657236D 01	0.3860483005D 01	-0.7100935759D 00	1	
5	-1.4	-0.1477247203D 01	0.5023548467D 00	-0.3362777351D 00	0.1125706193D 01	0.8919991439D 00	0.4460708690D 01	0.3334722687D 01	6	*
6	-1.0	-0.1059309762D 01	0.3737138238D 00	-0.1825945561D 00	0.6647362132D 00	0.6387900781D 00	0.4759606106D 01	0.3503732551D 00	4	*
7	-0.6	-0.6307806842D 00	0.2825549428D 00	-0.7342162599D-01	0.3302287040D 00	0.2389677362D 00	0.4136271163D 01	0.1235611924D 00	4	*
8	-0.2	-0.2092595422D 00	0.2366765956D 00	-0.2313937068D-01	0.1965938020D 00	0.4038658629D-01	0.3509612712D 01	0.3423139934D-01	4	*
9	0.2	0.2076874125D 00	0.2207286496D 00	0.1346050721D-02	0.1623264734D 00	0.1684793707D-03	0.3331746430D 01	0.1911230286D-03	4	**
10	0.6	0.6239177526D 00	0.2261937849D 00	0.2198074469D-01	0.1755324227D 00	0.4174868499D-01	0.3430804821D 01	0.4310691873D-01	4	**
11	1.0	0.1042490144D 01	0.2527129661D 00	0.5206805297D-01	0.2405490498D 00	0.1679809762D 00	0.3766592139D 01	0.1282889243D 00	4	*
12	1.4	0.1465767928D 01	0.3046277141D 00	0.1044900797D 00	0.3853276823D 00	0.3862263382D 00	0.4152325468D 01	0.2789693332D 00	4	*
13	1.8	0.1890412726D 01	0.3809518687D 00	0.1747823144D 00	0.5860594302D 00	0.5525666209D 00	0.4038326621D 01	0.1126837774D 01	6	*
14	2.2	0.2291372382D 01	0.4564356139D 00	0.2285810135D 00	0.7116711176D 00	0.5494671561D 00	0.3416019130D 01	-0.5764762882D 00	1	
15	2.6	0.2607935891D 01	0.4929310769D 00	0.2769887574D 00	0.6954443299D 00	0.6405686595D 00	0.2862134063D 01	-0.2628767387D 00	1	
16	3.0	0.2764415703D 01	0.5113279187D 00	0.4114256894D 00	0.6797516327D 00	0.1266147474D 01	0.2599867694D 01	-0.3269885102D 00	1	

Table 4-3 (Continued) 80 Items.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type	
1	-3.0	-0.2544633344D 01	0.7945317767D 00	-0.1220613203D 01	0.2627295467D 01	0.2970450456D 01	0.4161849527D 01	-0.7474169649D 00	1	
2	-2.6	-0.2497159606D 01	0.6626726791D 00	-0.7851874440D 00	0.1916118644D 01	0.2118602571D 01	0.4363392343D 01	-0.7130504683D 00	1	
3	-2.2	-0.2261945440D 01	0.5370988455D 00	-0.4963904043D 00	0.1407817754D 01	0.1590318454D 01	0.4880204265D 01	-0.1656359733D 01	1	
4	-1.8	-0.1881169117D 01	0.3769718793D 00	-0.2693637590D 00	0.8066803471D 00	0.1354414093D 01	0.5676538231D 01	0.1060075610D 01	6	*
5	-1.4	-0.1452359535D 01	0.2427809598D 00	-0.1039422429D 00	0.3105288636D 00	0.7549884295D 00	0.5268327031D 01	0.3020064591D 00	4	*
6	-1.0	-0.1028637569D 01	0.1679590482D 00	-0.3440387993D-01	0.1117009158D 00	0.2498070160D 00	0.3959587313D 01	0.1713794882D 00	4	*
7	-0.6	-0.6141049964D 00	0.1307792836D 00	-0.1274253358D-01	0.5759775241D-01	0.7259299453D-01	0.3367655272D 01	0.1072872084D 00	4	**
8	-0.2	-0.2042112360D 00	0.1125522486D 00	-0.4383698144D-02	0.4033752472D-01	0.1347779217D-01	0.3184204066D 01	0.3094610243D-01	4	**
9	0.2	0.2037530462D 00	0.1057897873D 00	0.2741118159D-03	0.3510334422D-01	0.6346353832D-04	0.3136613483D 01	0.1744161100D-03	4	**
10	0.6	0.6114165764D 00	0.1077885672D 00	0.4317376154D-02	0.3677596514D-01	0.1488406917D-01	0.3165327718D 01	0.3919753272D-01	4	**
11	1.0	0.1020055185D 01	0.1182393468D 00	0.9692725415D-02	0.4567382378D-01	0.5683367691D-01	0.3266956323D 01	0.1190585739D 00	4	**
12	1.4	0.1430988740D 01	0.1387499908D 00	0.1913703662D-01	0.6698305439D-01	0.1371041140D 00	0.3479357234D 01	0.1946334012D 00	4	**
13	1.8	0.1845920523D 01	0.1734072772D 00	0.3878328278D-01	0.1172622204D 00	0.2884610474D 00	0.3899630649D 01	0.2495131010D 00	4	*
14	2.2	0.2266883530D 01	0.2296564121D 00	0.8125557220D-01	0.2344642850D 00	0.5450929672D 00	0.4445489065D 01	0.3725874764D 00	4	*
15	2.6	0.2691597485D 01	0.3122551806D 00	0.1519283174D 00	0.4317873165D 00	0.7581384074D 00	0.4428438076D 01	0.1163022387D 01	6	*
16	3.0	0.3094765726D 01	0.4018798601D 00	0.2225493908D 00	0.5999442991D 00	0.7630694956D 00	0.3714654670D 01	-0.7957727218D 00	1	*

Table 4-3 (Continued) 120 Items.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type	
1	-3.0	-0.2809055138D 01	0.7145751288D 00	-0.1010276004D 01	0.2400706125D 01	0.2797282874D 01	0.4701573264D 01	-0.7983836231D 00	1	
2	-2.6	-0.2636695308D 01	0.5666254296D 00	-0.6118452090D 00	0.1663823995D 01	0.2057761054D 01	0.5182213014D 01	-0.1308106380D 01	1	
3	-2.2	-0.2285895226D 01	0.3924160574D 00	-0.3249635374D 00	0.9420108537D 00	0.1747547370D 01	0.6117336442D 01	0.1904035104D 01	6	*
4	-1.8	-0.1859584493D 01	0.2377325123D 00	-0.1207780616D 00	0.3374699942D 00	0.1085701332D 01	0.5971150317D 01	0.3943882919D 00	4	*
5	-1.4	-0.1433482675D 01	0.1504433032D 00	-0.3601865136D-01	0.9715782093D-01	0.3810099473D 00	0.4292714894D 01	0.2191262344D 00	4	*
6	-1.0	-0.1018369698D 01	0.1076002889D 00	-0.1267910526D-01	0.4011330029D-01	0.1290437285D 00	0.3464667164D 01	0.1845823017D 00	4	**
7	-0.6	-0.6091449873D 00	0.8528432481D-01	-0.5095028011D-02	0.2334464702D-01	0.4164904053D-01	0.3209584992D 01	0.1080736518D 00	4	**
8	-0.2	-0.2027294086D 00	0.7391583505D-01	-0.1808866755D-02	0.1701192850D-01	0.8102157682D-02	0.3113712953D 01	0.2998535045D-01	4	**
9	0.2	0.2024830317D 00	0.6960826246D-01	0.1135993091D-03	0.1495502713D-01	0.3826211745D-04	0.3086495293D 01	0.1660304779D-03	4	**
10	0.6	0.6075046338D 00	0.7081692864D-01	0.1791933534D-02	0.1556197686D-01	0.9041334395D-02	0.3103062983D 01	0.3797606437D-01	4	**
11	1.0	0.1013138233D 01	0.7731326070D-01	0.3963897770D-02	0.1888907585D-01	0.3400031341D-01	0.3160113858D 01	0.1178822354D 00	4	**
12	1.4	0.1420196811D 01	0.8990985131D-01	0.7583891205D-02	0.2643049016D-01	0.7913385995D-01	0.3269570139D 01	0.2007021958D 00	4	**
13	1.8	0.1829688238D 01	0.1106174687D 00	0.1454351954D-01	0.4244075553D-01	0.1562672210D 00	0.3468451883D 01	0.2604959645D 00	4	**
14	2.2	0.2243050086D 01	0.1433476612D 00	0.2948389804D-01	0.7918616362D-01	0.2951195406D 00	0.3853612833D 01	0.2902264890D 00	4	*
15	2.6	0.2662298395D 01	0.1952714556D 00	0.6367336137D-01	0.1698963474D 00	0.5445017295D 00	0.4455603276D 01	0.3658119949D 00	4	*
16	3.0	0.3087073903D 01	0.2740658317D 00	0.1289712729D 00	0.3489371343D 00	0.8078416004D 00	0.4644874857D 01	0.8434185557D 00	4	*

Table 4-3 (Continued) 200 Items.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type	
1	-3.0	-0.3019827604D 01	0.5837050349D 00	-0.7191763817D 00	0.1949663689D 01	0.2600701537D 01	0.5722329012D 01	-0.1390735734D 01	1	
2	-2.6	-0.2686835602D 01	0.3885397558D 00	-0.3572410967D 00	0.1024986243D 01	0.2175787308D 01	0.6789645671D 01	0.2402032437D 01	6	*
3	-2.2	-0.2261305892D 01	0.2199116777D 00	-0.1198248753D 00	0.3204102849D 00	0.1350047647D 01	0.6625365848D 01	0.4351628694D 00	4	*
4	-1.8	-0.1834165215D 01	0.1287512171D 00	-0.3138384316D-01	0.7412487837D-01	0.4614856697D 00	0.4471583111D 01	0.2503946728D 00	4	*
5	-1.4	-0.1419223772D 01	0.8549932056D-01	-0.1023087834D-01	0.2567198076D-01	0.1674703200D 00	0.3511834586D 01	0.2514519387D 00	4	**
6	-1.0	-0.1010717892D 01	0.6279845242D-01	-0.4048112768D-02	0.1273596788D-01	0.6616958033D-01	0.3229490955D 01	0.1937635169D 00	4	**
7	-0.6	-0.6053738506D 00	0.5033755839D-01	-0.1702879726D-02	0.7890602777D-02	0.2273481951D-01	0.3114052200D 01	0.1072605828D 00	4	**
8	-0.2	-0.2016026735D 00	0.4383977901D-01	-0.6163186699D-03	0.5889973302D-02	0.4508224957D-02	0.3064619874D 01	0.2925537272D-01	4	**
9	0.2	0.2014809303D 00	0.4134176143D-01	0.3867230034D-04	0.5212790944D-02	0.2116572174D-04	0.3049947439D 01	0.1590227019D-03	4	**
10	0.6	0.6044541545D 00	0.4201344471D-01	0.6125880435D-03	0.5399351503D-02	0.5060252159D-02	0.3058898167D 01	0.3703397542D-01	4	**
11	1.0	0.1007778404D 01	0.4571054759D-01	0.1342071096D-02	0.6454958406D-02	0.1885826674D-01	0.3089303670D 01	0.1164591716D 00	4	**
12	1.4	0.1411914494D 01	0.5282760165D-01	0.2518528693D-02	0.8777339733D-02	0.4302403348D-01	0.3145148238D 01	0.2023311519D 00	4	**
13	1.8	0.1817414827D 01	0.6432788836D-01	0.4666691608D-02	0.1339771722D-01	0.8181262322D-01	0.3237667279D 01	0.2724516536D 00	4	**
14	2.2	0.2225026977D 01	0.8194830226D-01	0.8901852820D-02	0.2277856542D-01	0.1439928531D 00	0.3391926616D 01	0.3182015966D 00	4	**
15	2.6	0.2635848791D 01	0.1087017188D 00	0.1788435306D-01	0.4337861173D-01	0.2490217240D 00	0.3671155887D 01	0.3339610346D 00	4	**
16	3.0	0.3051577609D 01	0.1500363722D 00	0.3867165310D-01	0.9461073018D-01	0.4427879675D 00	0.4202882852D 01	0.3442793805D 00	4	*

Table 4-4

Conditional Mean μ_1' , Second to Fourth Moments, μ_2 , μ_3 and μ_4 , Indices β_1 and β_2 , Pearson's Criterion κ and Pearson's Type of Distribution Obtained at Sixteen Different Values of θ , for Each of the Five Hypothetical Tests of Equivalent Items on the Three-Parameter Logistic Model with $a_g = 0.50$, $b_g = 0.00$ and $c_g = 0.25$. Moments Were Calculated by "Ignoring" Negative and Positive Infinities. The Mark ** Indicates That the Sum Total of the Probabilities for Finite Values of $\hat{\theta}_L$ Is Greater Than, or Equal to, 0.999999 , and * Means It Is Greater Than, or Equal to, 0.99 and Less Than 0.999999 . 20 Items.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type
1	-3.0	-0.1274142202D 01	0.8259047841D 00	-0.1292493057D 01	0.2298587707D 01	0.2965285351D 01	0.3369776646D 01	-0.8046147200D 00	1
2	-2.6	-0.1355360171D 01	0.7596647220D 00	-0.1063815542D 01	0.1891765071D 01	0.2581468239D 01	0.3278108206D 01	-0.6592196759D 00	1
3	-2.2	-0.1411431610D 01	0.7117325804D 00	-0.8240863755D 00	0.1599481961D 01	0.1883627092D 01	0.3157516678D 01	-0.4794440684D 00	1
4	-1.8	-0.1400376380D 01	0.7045398146D 00	-0.6032605890D 00	0.1509876767D 01	0.1040622676D 01	0.3041798358D 01	-0.3455532639D 00	1
5	-1.4	-0.1275352628D 01	0.7257846987D 00	-0.4406461780D 00	0.1606317093D 01	0.5078747144D 00	0.3049408909D 01	-0.3055195338D 00	1
6	-1.0	-0.1014494476D 01	0.7236029821D 00	-0.3522480675D 00	0.1721702109D 01	0.3274886059D 00	0.3288193113D 01	-0.6550512194D 00	1
7	-0.6	-0.6442050555D 00	0.6680363853D 00	-0.2746265175D 00	0.1654076421D 01	0.2529790674D 00	0.3706426020D 01	0.3092369565D 00	4
8	-0.2	-0.2198328657D 00	0.5921550824D 00	-0.1478253684D 00	0.1394697245D 01	0.1052425971D 00	0.3977489306D 01	0.5010956065D-01	4 *
9	0.2	0.2175303247D 00	0.5471989600D 00	-0.3975118167D-02	0.1176740085D 01	0.9644157512D-04	0.3929977099D 01	0.3960875545D-04	4 *
10	0.6	0.6532510586D 00	0.5501741911D 00	0.1143505799D 00	0.1135685066D 01	0.7851924265D-01	0.3751953849D 01	0.4776272137D-01	4 *
11	1.0	0.1077792142D 01	0.5809693790D 00	0.1863884227D 00	0.1148192930D 01	0.1771650855D 00	0.3401796914D 01	0.5101888649D 00	4 *
12	1.4	0.1463506412D 01	0.5986771244D 00	0.2073026780D 00	0.1051465197D 01	0.2002773485D 00	0.2933658616D 01	-0.2158562139D 00	1
13	1.8	0.1759588524D 01	0.5736197266D 00	0.2329915862D 00	0.8483791224D 00	0.2876128209D 00	0.2578349677D 01	-0.1387674878D 00	1
14	2.2	0.1913839057D 01	0.5338348472D 00	0.3400727832D 00	0.6829248936D 00	0.7601914692D 00	0.2396398858D 01	-0.2172205667D 00	1
15	2.6	0.1905449378D 01	0.5665528695D 00	0.5575994269D 00	0.7526303099D 00	0.1709713365D 01	0.2344773068D 01	-0.4461483886D 00	1
16	3.0	0.1757663679D 01	0.7331758079D 00	0.9298982108D 00	0.1252020382D 01	0.2194049619D 01	0.2329137598D 01	-0.7189529825D 00	1

Table 4-4 (Continued) 40 Items.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type
1	-3.0	-0.1838728208D 01	0.9195818771D 00	-0.1543056044D 01	0.3034008235D 01	0.3061908364D 01	0.3587863797D 01	-0.8028923678D 00	1
2	-2.6	-0.1921753750D 01	0.7910604823D 00	-0.1137356708D 01	0.2278788439D 01	0.2613149866D 01	0.3641536171D 01	-0.6533948898D 00	1
3	-2.2	-0.1918428444D 01	0.7133649072D 00	-0.8002360399D 00	0.1843675543D 01	0.1764009785D 01	0.3622938921D 01	-0.5196673704D 00	1
4	-1.8	-0.1770976093D 01	0.6605190403D 00	-0.5649948837D 00	0.1632256185D 01	0.1107727676D 01	0.3741257845D 01	-0.5872968190D 00	1
5	-1.4	-0.1467521253D 01	0.5682986636D 00	-0.4046019422D 00	0.1365140022D 01	0.8919202457D 00	0.4226919040D 01	-0.3687290988D 01	1
6	-1.0	-0.1064935139D 01	0.4369672153D 00	-0.2445760132D 00	0.9011910046D 00	0.7169372745D 00	0.4719751574D 01	0.4954842400D 00	4 *
7	-0.6	-0.6355575162D 00	0.3279276448D 00	-0.1071166585D 00	0.4701595799D 00	0.3253718250D 00	0.4372091374D 01	0.1514245631D 00	4 *
8	-0.2	-0.2107538568D 00	0.2684038235D 00	-0.3452412197D-01	0.2636467214D 00	0.6164235043D-01	0.3659695725D 01	0.4168246384D-01	4 **
9	0.2	0.2084366708D 00	0.2462118759D 00	-0.3094407725D-03	0.2056101676D 00	0.6415464141D-05	0.3391771549D 01	0.6164138516D-05	4 **
10	0.6	0.6263513444D 00	0.2499116519D 00	0.2608470752D-01	0.2177173811D 00	0.4359256535D-01	0.3485941472D 01	0.3946031093D-01	4 *
11	1.0	0.1046547008D 01	0.2777870542D 00	0.6270247285D-01	0.2956843479D 00	0.1834143582D 00	0.3831813215D 01	0.1300810464D 00	4 *
12	1.4	0.1471104392D 01	0.3328761486D 00	0.1217600978D 00	0.4580547473D 00	0.4019406609D 00	0.4133824477D 01	0.3141707362D 00	4 *
13	1.8	0.1893909829D 01	0.4094884336D 00	0.1908376695D 00	0.6516566018D 00	0.5303997261D 00	0.3886292827D 01	0.2484342783D 01	6 *
14	2.2	0.2283737293D 01	0.4765154452D 00	0.2368071282D 00	0.7390453080D 00	0.5182730143D 00	0.3254745988D 01	-0.4229834148D 00	1
15	2.6	0.2575485925D 01	0.5009404961D 00	0.2907001313D 00	0.6953140197D 00	0.6722518889D 00	0.2770822484D 01	-0.2494086636D 00	1
16	3.0	0.2698130764D 01	0.5223528271D 00	0.4468984851D 00	0.6975446807D 00	0.1401283203D 01	0.2556490199D 01	-0.3527981676D 00	1

Table 4-4 (Continued) 80 Items.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type
1	-3.0	-0.2398465923D 01	0.8597562049D 00	-0.1416393637D 01	0.3074535667D 01	0.3156763002D 01	0.4159382861D 01	-0.7891917016D 00	1
2	-2.6	-0.2397579156D 01	0.7191738202D 00	-0.9405736437D 00	0.2236973473D 01	0.2378395099D 01	0.4325069545D 01	-0.6997924320D 00	1
3	-2.2	-0.2224171369D 01	0.6042140879D 00	-0.6106096991D 00	0.1700661035D 01	0.1690265221D 01	0.4658392385D 01	-0.1041811831D 01	1
4	-1.8	-0.1883256201D 01	0.4505735982D 00	-0.3635240647D 00	0.1104847784D 01	0.1444671519D 01	0.5442155772D 01	0.2682912481D 01	6 *
5	-1.4	-0.1461503897D 01	0.2954615114D 00	-0.1595289586D 00	0.4904892079D 00	0.9866799773D 00	0.5618593677D 01	0.4123293391D 00	4 *
6	-1.0	-0.1033730718D 01	0.1982092440D 00	-0.5305650326D-01	0.1711648446D 00	0.3614976952D 00	0.4356791332D 01	0.1837191303D 00	4 *
7	-0.6	-0.6162447044D 00	0.1498748133D 00	-0.1833129090D-01	0.7837810802D-01	0.9981599331D-01	0.3489293211D 01	0.1132917442D 00	4 **
8	-0.2	-0.2047893849D 00	0.1265542708D 00	-0.6223041294D-02	0.5163190151D-01	0.1910622767D-01	0.3223773405D 01	0.3693296510D-01	4 **
9	0.2	0.2041355639D 00	0.1173876637D 00	-0.4743565027D-04	0.4348059325D-01	0.1391048716D-05	0.3155371821D 01	0.3359573308D-05	4 **
10	0.6	0.6125353634D 00	0.1184997477D 00	0.4972076656D-02	0.4467799202D-01	0.1485673423D-01	0.3181696827D 01	0.3510211820D-01	4 **
11	1.0	0.1021894038D 01	0.1291923755D 00	0.1143417411D-01	0.5496287657D-01	0.6063158903D-01	0.3293029994D 01	0.1143359864D 00	4 **
12	1.4	0.1433694292D 01	0.1511045205D 00	0.2275381845D-01	0.8068064130D-01	0.1500638918D 00	0.3533575925D 01	0.1896899612D 00	4 **
13	1.8	0.1849804622D 01	0.1887484573D 00	0.4638671453D-01	0.1423726616D 00	0.3199904310D 00	0.3996315619D 01	0.2523687390D 00	4 *
14	2.2	0.2272076331D 01	0.2497654879D 00	0.9548590570D-01	0.2796250364D 00	0.5851689300D 00	0.4482406066D 01	0.4187427012D 00	4 *
15	2.6	0.2695666515D 01	0.3356911496D 00	0.1688985787D 00	0.4835979400D 00	0.7541058603D 00	0.4291455955D 01	0.2097763611D 01	6 *
16	3.0	0.3088036593D 01	0.4205512345D 00	0.2353118003D 00	0.6264412698D 00	0.7444416733D 00	0.3541950903D 01	-0.5806331570D 00	1

Table 4-4 (Continued) 120 Items.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type
1	-3.0	-0.2684525359D 01	0.7849843678D 00	-0.1220524491D 01	0.2868823041D 01	0.3079711053D 01	0.4655665222D 01	-0.8112523444D 00	1
2	-2.6	-0.2579263044D 01	0.6377086049D 00	-0.7638071356D 00	0.2029676857D 01	0.2249575591D 01	0.4990939961D 01	-0.9821616212D 00	1
3	-2.2	-0.2282235651D 01	0.4721747306D 00	-0.4415049665D 00	0.1293090155D 01	0.1851667325D 01	0.5799937621D 01	0.4527505741D 02	6
4	-1.8	-0.1870300041D 01	0.2966724104D 00	-0.1906887091D 00	0.5560985450D 00	0.1392574623D 01	0.6318259199D 01	0.5828004105D 00	4 *
5	-1.4	-0.1440168297D 01	0.1821029418D 00	-0.5837080631D-01	0.1618961210D 00	0.5642096372D 00	0.4882053339D 01	0.2371874065D 00	4 *
6	-1.0	-0.1021467690D 01	0.1256572840D 00	-0.1872601948D-01	0.5751612166D-01	0.1767371862D 00	0.3642623264D 01	0.1839095612D 00	4 **
7	-0.6	-0.6104666623D 00	0.9729872315D-01	-0.7144189844D-02	0.3090217245D-01	0.5540954069D-01	0.3264184736D 01	0.1164410316D 00	4 **
8	-0.2	-0.2030846812D 00	0.8292920032D-01	-0.2536502063D-02	0.2156414969D-01	0.1128101060D-01	0.3135576370D 01	0.3576673634D-01	4 **
9	0.2	0.2027390362D 00	0.7712800099D-01	-0.2071192900D-04	0.1842710369D-01	0.9349854897D-06	0.3097654159D 01	0.3591394659D-05	4 **
10	0.6	0.6082324790D 00	0.7774886355D-01	0.2047879082D-02	0.1981476615D-01	0.8923320072D-02	0.3112509785D 01	0.3384210566D-01	4 **
11	1.0	0.1014323068D 01	0.8433074439D-01	0.4635333828D-02	0.2256988622D-01	0.3582648755D-01	0.3173638836D 01	0.1130982521D 00	4 **
12	1.4	0.1421918267D 01	0.9766099920D-01	0.8905959779D-02	0.3441684147D-01	0.8515261093D-01	0.3293974204D 01	0.1963049656D 00	4 **
13	1.8	0.1832126992D 01	0.1199149347D 00	0.1715162240D-01	0.5058494047D-01	0.1706045416D 00	0.3517328740D 01	0.2550885205D 00	4 **
14	2.2	0.2246524827D 01	0.1554578643D 00	0.3504866662D-01	0.9551022433D-01	0.3269677432D 00	0.3952068568D 01	0.2886005209D 00	4 *
15	2.6	0.2667126986D 01	0.2120460768D 00	0.7527325390D-01	0.2040633279D 00	0.5942793712D 00	0.4538417882D 01	0.3985536618D 00	4 *
16	3.0	0.3091778821D 01	0.2955599102D 00	0.1457203029D 00	0.3972381538D 00	0.8224387289D 00	0.4547365924D 01	0.1187318279D 01	6 *

Table 4-4 (Continued) 200 Items.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type	
1	-3.0	-0.2948729075D 01	0.6646286102D 00	-0.9172049363D 00	0.2437106683D 01	0.2865468704D 01	0.5517171396D 01	-0.1082888479D 01	1	
2	-2.6	-0.2679829188D 01	0.4753691881D 00	-0.4999192177D 00	0.1449886676D 01	0.2326518861D 01	0.6416112959D 01	-0.1873294638D 02	1	
3	-2.2	-0.2273351560D 01	0.2821797616D 00	-0.1991349869D 00	0.5645316353D 00	0.1764889637D 01	0.7089842045D 01	0.6750413826D 00	4	*
4	-1.8	-0.1841799699D 01	0.1596954841D 00	-0.5358153830D-01	0.1339758650D 00	0.7049404795D 00	0.5253410032D 01	0.2655606464D 00	4	*
5	-1.4	-0.1422847948D 01	0.1018299096D 00	-0.1557912692D-01	0.3858839330D-01	0.2298582819D 00	0.3721396798D 01	0.2427905301D 00	4	**
6	-1.0	-0.1012441478D 01	0.7285120867D-01	-0.5780152890D-02	0.1748531216D-01	0.8641086799D-01	0.3294578553D 01	0.2008146901D 00	4	**
7	-0.6	-0.6061242348D 00	0.5726003958D-01	-0.2353802260D-02	0.1029579482D-01	0.2951107428D-01	0.3140194809D 01	0.1162434197D 00	4	**
8	-0.2	-0.2018029370D 00	0.4910797894D-01	-0.8573591139D-03	0.7418568817D-02	0.6206822916D-02	0.3076210200D 01	0.3484983719D-01	4	**
9	0.2	0.2016349013D 00	0.4575850170D-01	-0.7552545117D-05	0.6399018931D-02	0.5953485081D-06	0.3056115782D 01	0.3978888586D-05	4	**
10	0.6	0.6048828838D 00	0.4607973354D-01	0.6962711292D-03	0.6505966243D-02	0.4954808772D-02	0.3064022057D 01	0.3287722252D-01	4	**
11	1.0	0.1008471524D 01	0.4979900063D-01	0.1560286462D-02	0.7678369543D-02	0.1971273114D-01	0.3096191080D 01	0.1115186255D 00	4	**
12	1.4	0.1412913038D 01	0.5728290596D-01	0.2935409146D-02	0.1035761406D-01	0.4584185270D-01	0.3156527966D 01	0.1981565023D 00	4	**
13	1.8	0.1818810813D 01	0.6954706857D-01	0.5441246379D-02	0.1575612738D-01	0.8801572994D-01	0.3257555510D 01	0.2688219870D 00	4	**
14	2.2	0.2226977956D 01	0.8847877618D-01	0.1040257932D-01	0.2684049365D-01	0.1562301145D 00	0.3428564193D 01	0.3137187221D 00	4	**
15	2.6	0.2638613464D 01	0.1174181297D 00	0.2104243711D-01	0.5164295238D-01	0.2735184339D 00	0.3745766886D 01	0.3274465241D 00	4	**
16	3.0	0.3055549815D 01	0.1624660541D 00	0.4586947787D-01	0.1141711258D 00	0.4906363214D 00	0.4325447234D 01	0.3526808722D 00	4	*

$$(4.2) \qquad \mu_k = E(\{\hat{\theta}_t - \mu_1'\}^k | \theta) \quad ,$$

for $k = 2,3,4$. In the same table, also presented are the indices β_1 and β_2 and Pearson's criterion κ , which are defined by

$$(4.3) \qquad \beta_1 = \mu_3^2 \ \mu_2^{-3} \quad ,$$

$$(4.4) \qquad \beta_2 = \mu_4 \ \mu_2^{-2}$$

and

$$(4.5) \qquad \kappa = \beta_1(\beta_2+3)^2[4(2\beta_2-3\beta_1-6)(4\beta_2-3\beta_1)]^{-1} \quad .$$

This criterion κ is used to determine which type of Pearson's distributions should be chosen to fit our empirical distribution (cf. Elderton and Johnson, 1969; Johnson and Kotz, 1970), and those types are given in the last columns of Tables 4-2 through 4-4.

We realize that those moments and indices cannot seriously be taken into consideration unless the two probabilities assigned to the negative and positive infinities, which are presented in Table 4-1, are negligibly small. Those cases are indicated in Tables 4-2 through 4-4 by ** and * , the former of which indicates that the sum total of the probabilities assigned to the finite values of $\hat{\theta}_t$ is greater than, or equal to, 0.999999 , and the latter means that it is greater than, or equal to, 0.99 but less than 0.999999 .

If a distribution is normal, then we will have $\beta_1 = 0.00$, $\beta_2 = 3.00$ and the criterion κ converges to zero. We anticipate that, since those five hypothetical tests consist of equivalent items, in the normal ogive model the convergence to the normality must be speediest at $\theta = b_g$, and in the three-parameter logistic model it must be fastest at

$$(4.6) \qquad \theta = b_g + (Da_g)^{-1} \log[\{1-4c_g+(1+8c_g)^{1/2}\}\{3-(1+8c_g)^{1/2}\}^{-1}] \quad ,$$

at which the common item information function $I_g(\theta)$ assumes the highest value. With our hypothetical tests of equivalent items, those values are 0.00 for the normal ogive model, 0.31428 for the three-parameter logistic model with $c_g = 0.20$, and 0.36695 for the three-parameter logistic model with $c_g = 0.25$.

Comparison of those three tables indicates that there exist substantial differences in the convergence to the normality of the conditional distribution of $\hat{\theta}_t$, given θ , between the results on the normal ogive model and those on the three-parameter logistic model. For example, for $n = 200$, the result on the normal ogive model provides us with the regression which differs from θ by less than 0.01 in absolute value, β_1 which is less than 0.01 , β_2 which differs from 3.00 by less than 0.10 in absolute value, and κ which is less than 0.10 in absolute value, for as many as six points of θ , i.e., -1.0 , -0.6 , -0.2 , 0.2 , 0.6 and 1.0 , whereas the same is true for the results on the three-parameter logistic model only for three points of θ , i.e., -0.2 , 0.2 and 0.6 in each of the two cases where $c_g = 0.20$ and

$c_g = 0.25$, respectively. As another example, for $n = 80$, the above is true for as many as four points of θ , i.e., -0.6 , -0.2 , 0.2 and 0.6 , on the normal ogive model, while at no point of θ is it satisfied on the three-parameter logistic model in either of the two cases where $c_g = 0.20$ and $c_g = 0.25$. If we relax this rule to $|\mu_1' - \theta| < 0.03$, $\beta_1 < 0.05$, $|\beta_2 - 3| < 0.50$, and $|\kappa| < 0.25$, then the resultant number of points at which this condition is satisfied is as shown in Table 4-5 for each test and each model. From this table, too, we can see an obvious effect of noise caused by random guessing in the three-parameter logistic model on the speed of convergence of the conditional distribution of $\hat{\theta}_t$, given θ , to the normality.

Figures 4-1 through 4-3 present μ_1' , that is the regression of $\hat{\theta}_t$ on θ , and the confidence interval $(\mu_1' - \mu_2^{1/2}$, $\mu_1' + \mu_2^{1/2})$ plotted against the sixteen points of θ , together with the asymptotic, unbiased regression with the confidence interval $(\theta - \{I(\theta)\}^{-1/2}, \theta + \{I(\theta)\}^{-1/2})$, which were first presented in Figure 3-2 of the preceding section. Since those moments were computed by "ignoring" negative and positive infinities, we cannot take some sets of three points seriously if the probabilities assigned to either negative or positive infinities, or both, are substantially large. For this reason, in Figures 4-1 through 4-3, five different symbols are used to indicate the magnitude of the sum total of the probabilities assigned to the finite values of $\hat{\theta}_t$, which are shown in Table 4-6. The S shape observed in some of the graphs in those three figures is caused by smaller values of the sum total of those probabilities. In those graphs, when this sum is greater than, or equal

TABLE 4-5

Number of Points of θ at which $|\mu_1' - \theta| < 0.03$, $\beta_1 < 0.05$, $|\beta_2 - 3| < 0.50$ and $\kappa < 0.25$, for Each of the Five Tests of Equivalent Items Following the Normal Ogive Model and the Three-Parameter Logistic Model with $c_g = 0.20$ and $c_g = 0.25$.

n	Normal Ogive	3-P.L. c_g=0.20	3-P.L. c_g=0.25
20	4	0	0
40	6	2	2
80	8	3	3
120	8	5	4
200	10	6	6

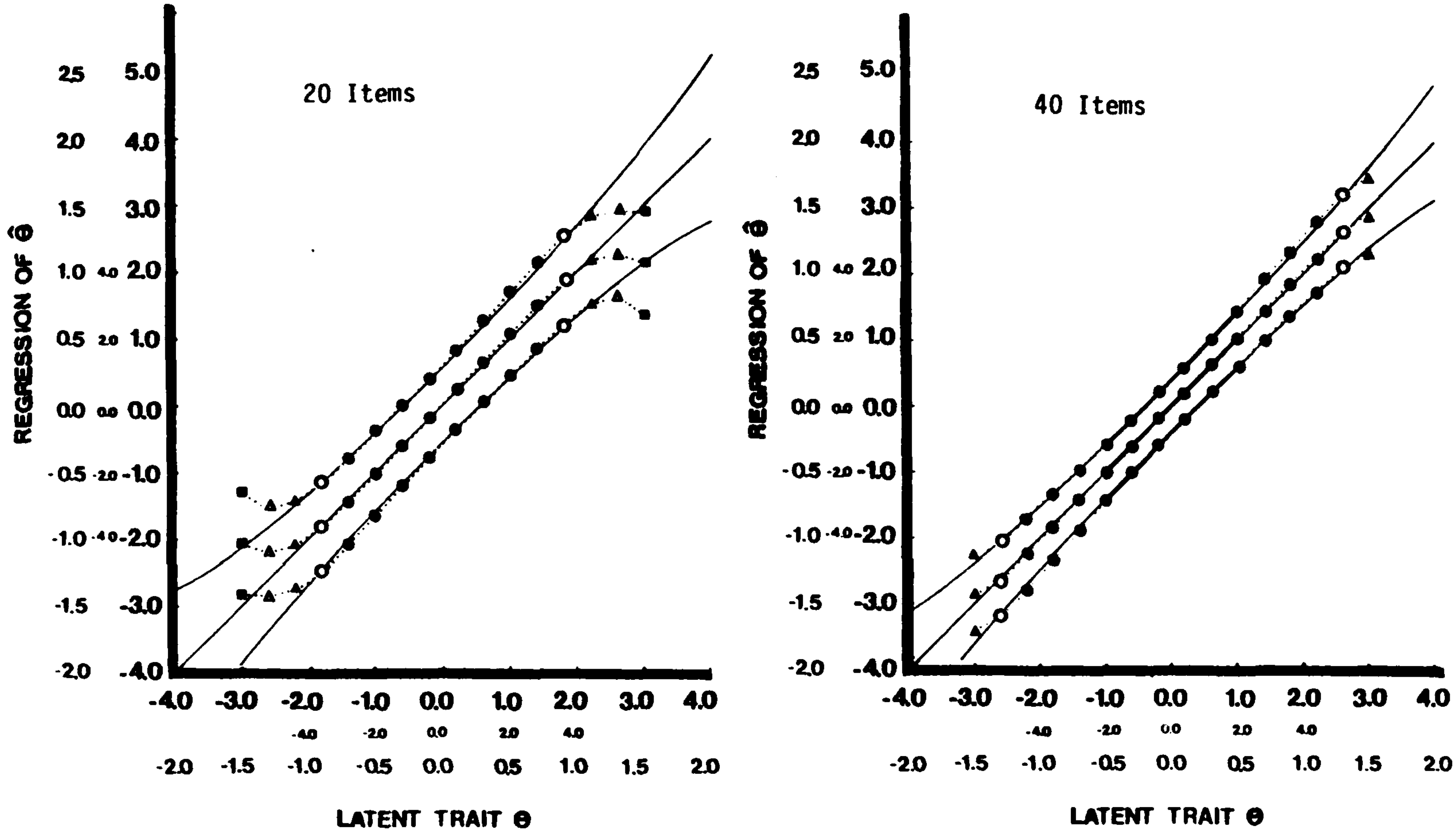


FIGURE 4-1

Regression of $\hat{\theta}_t$ on θ , μ_1' , and the Confidence Interval $(\mu_1'-\mu_2^{1/2}, \mu_1'+\mu_2^{1/2})$ Plotted against the Sixteen Points of θ , Together with the Asymptotic, Unbiased Regression and the Confidence Interval $(\theta-\{I(\theta)\}^{-1/2}, \theta+\{I(\theta)\}^{-1/2})$, for Each of the Five Tests of Equivalent Items on the Normal Ogive Model with $a_g = 0.50$ and $b_g = 0.00$.

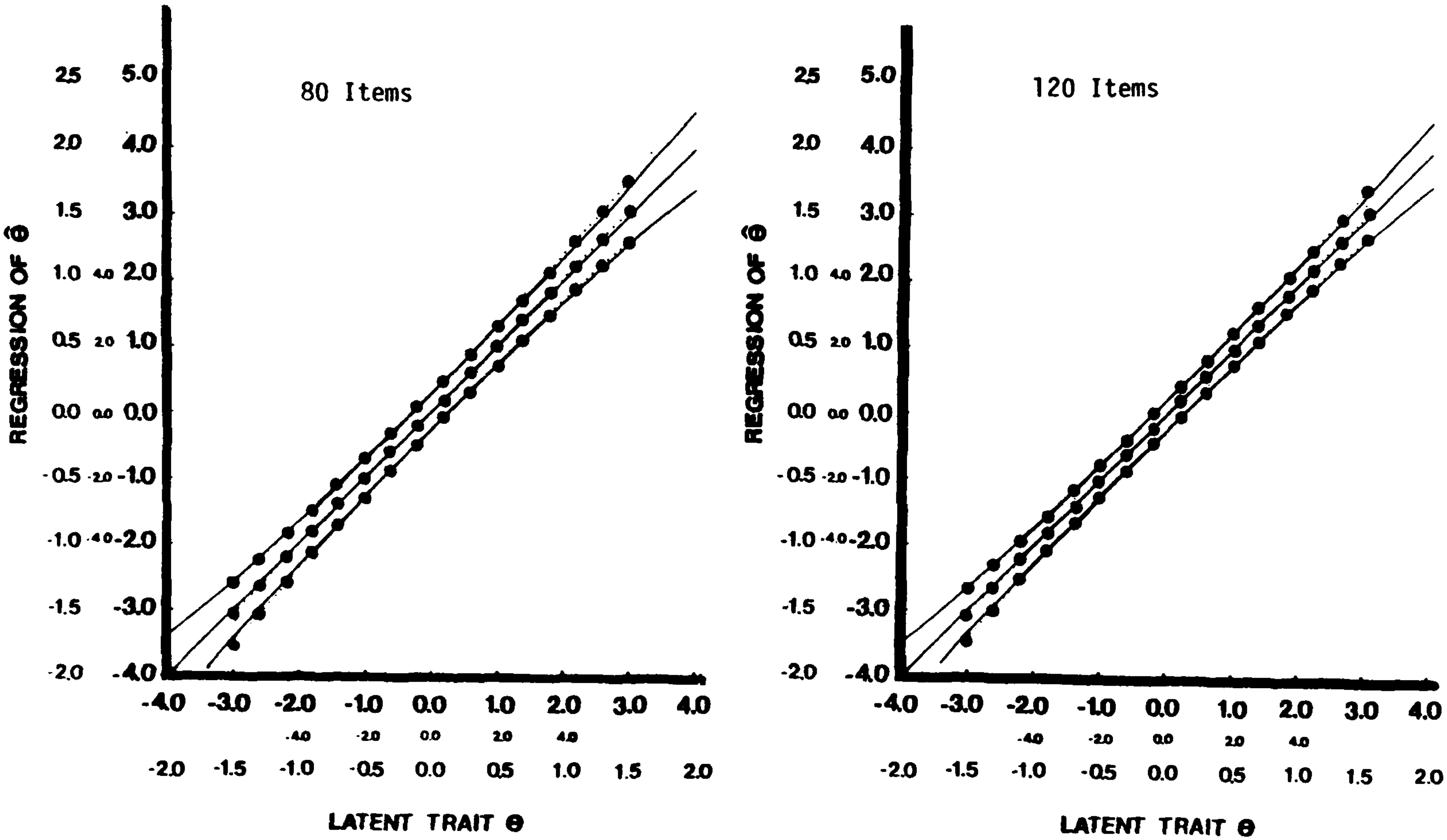


FIGURE 4-1 (Continued)

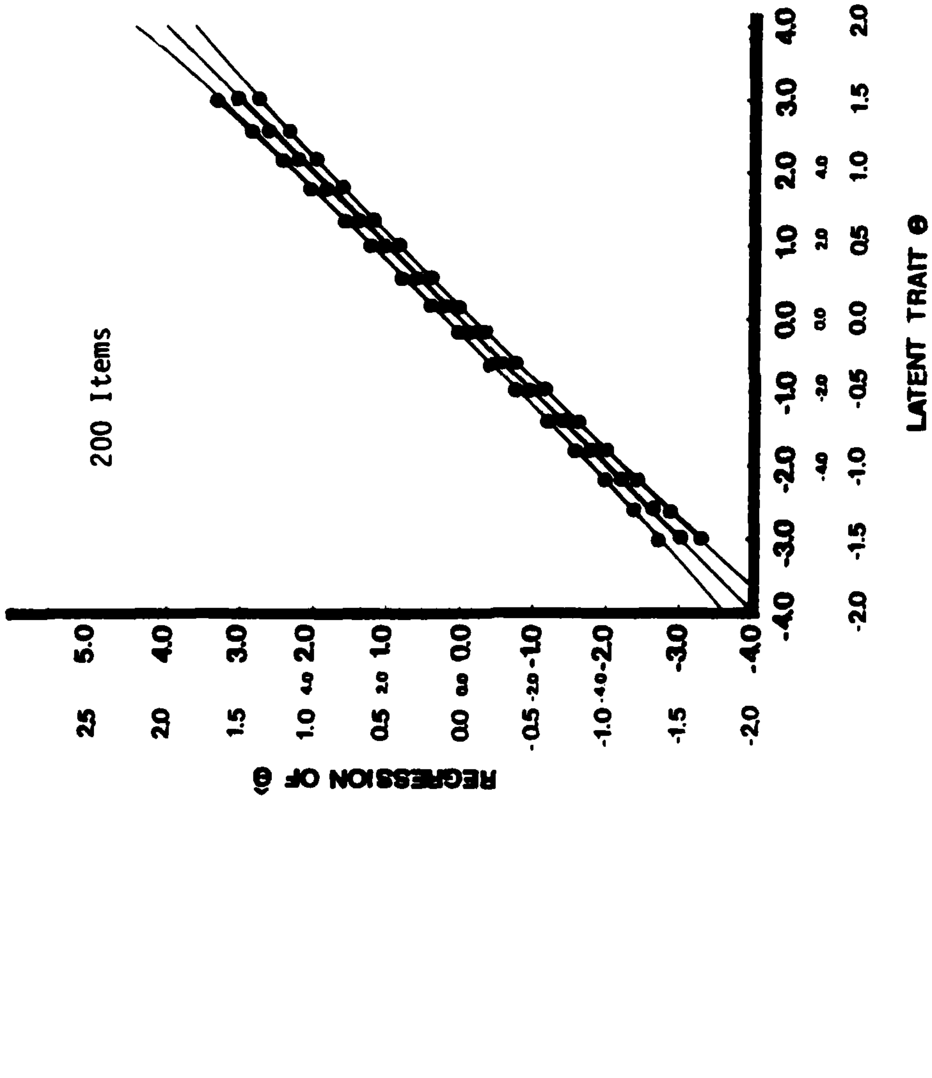


FIGURE 4-1 (Continued)

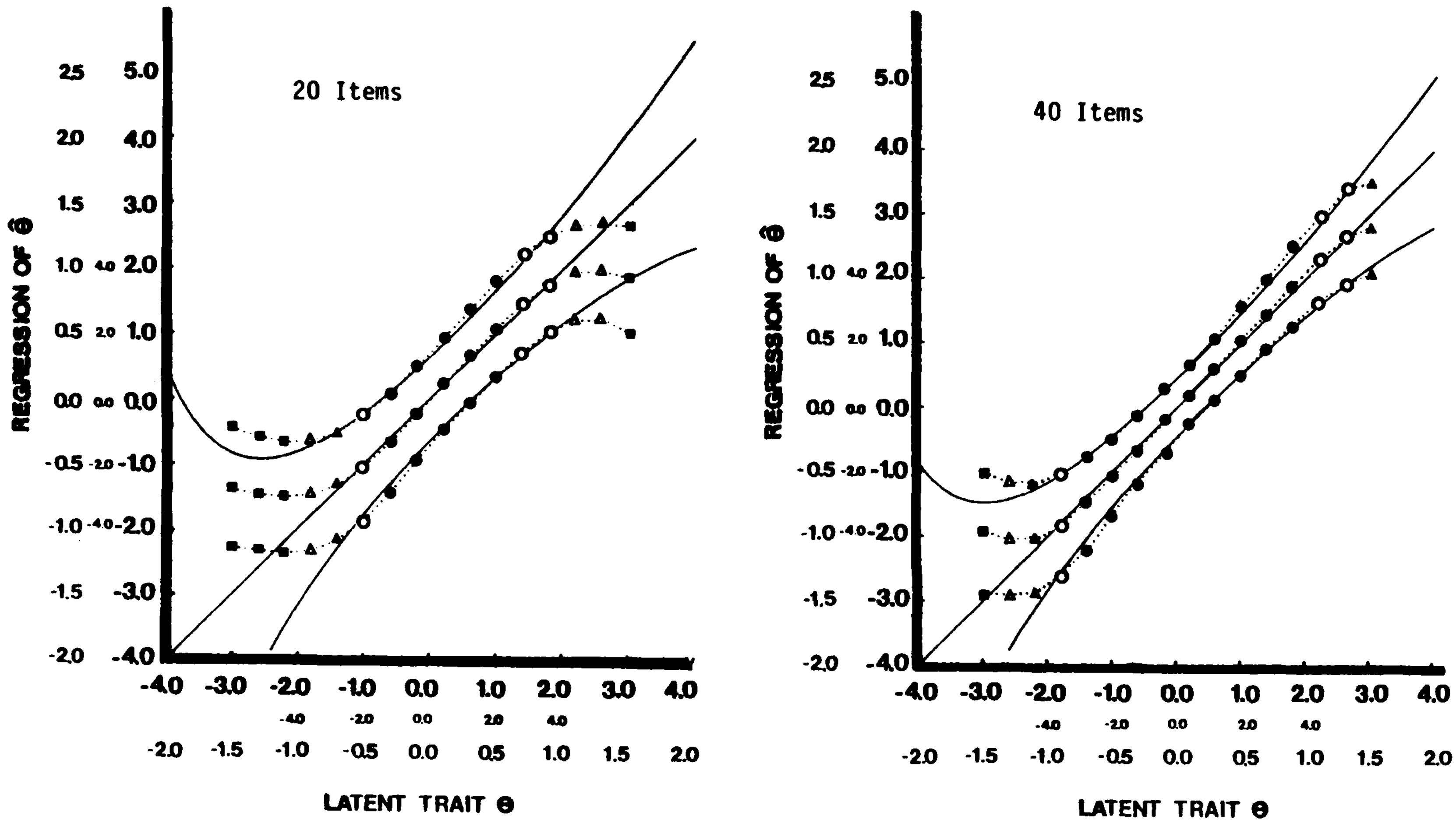


FIGURE 4-2

Regression of $\hat{\theta}_t$ on θ , μ_1' , and the Confidence Interval $(\mu_1'-\mu_2^{1/2}, \mu_1'+\mu_2^{1/2})$ Plotted against the Sixteen Points of θ , Together with the Asymptotic, Unbiased Regression and the Confidence Interval $(\theta-\{I(\theta)\}^{-1/2}, \theta+\{I(\theta)\}^{-1/2})$, for Each of the Five Tests of Equivalent Items on the Three-Parameter Logistic Model with $a_g = 0.50$, $b_g = 0.00$ and $c_g = 0.20$.

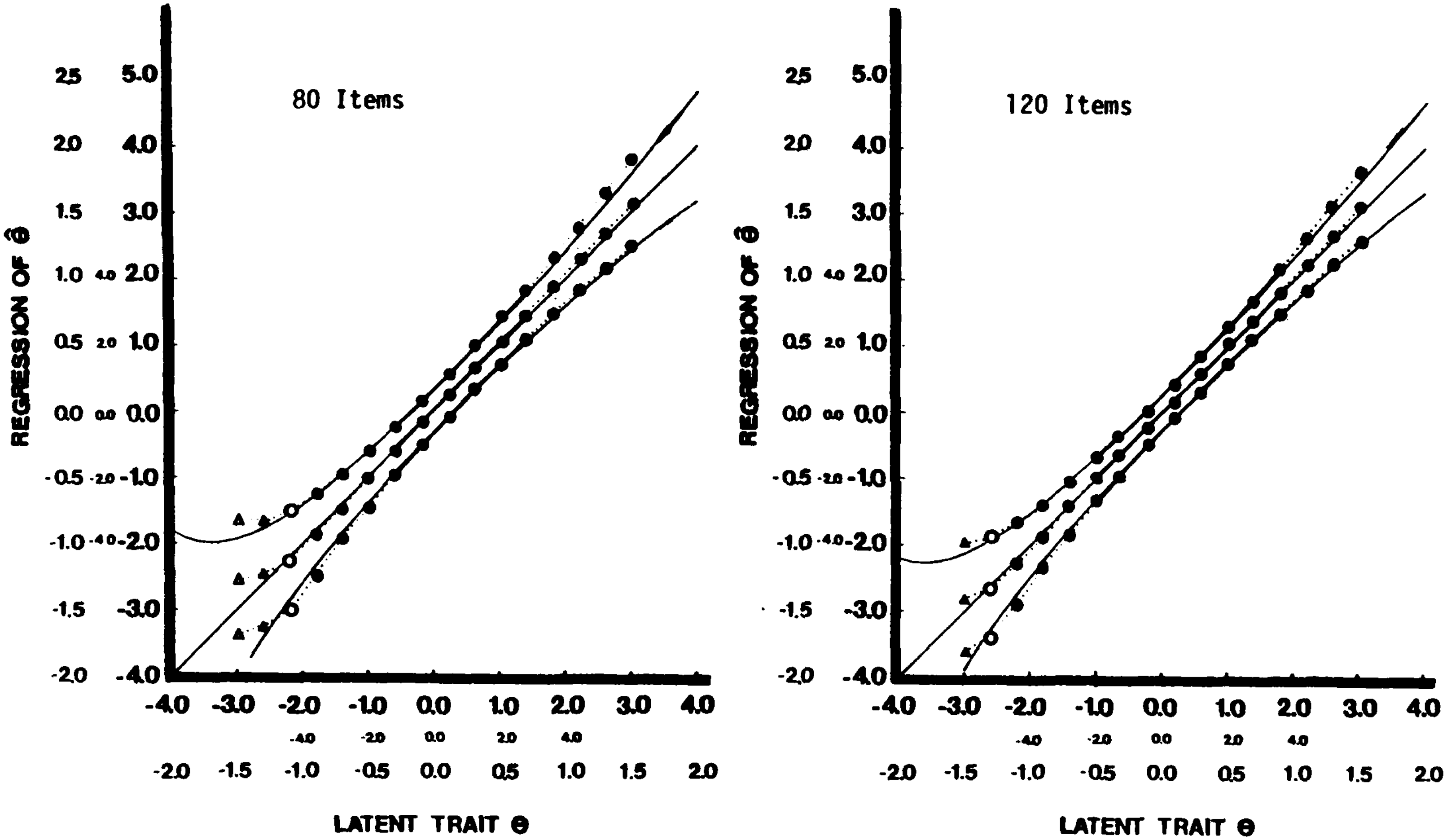


FIGURE 4-2 (Continued)

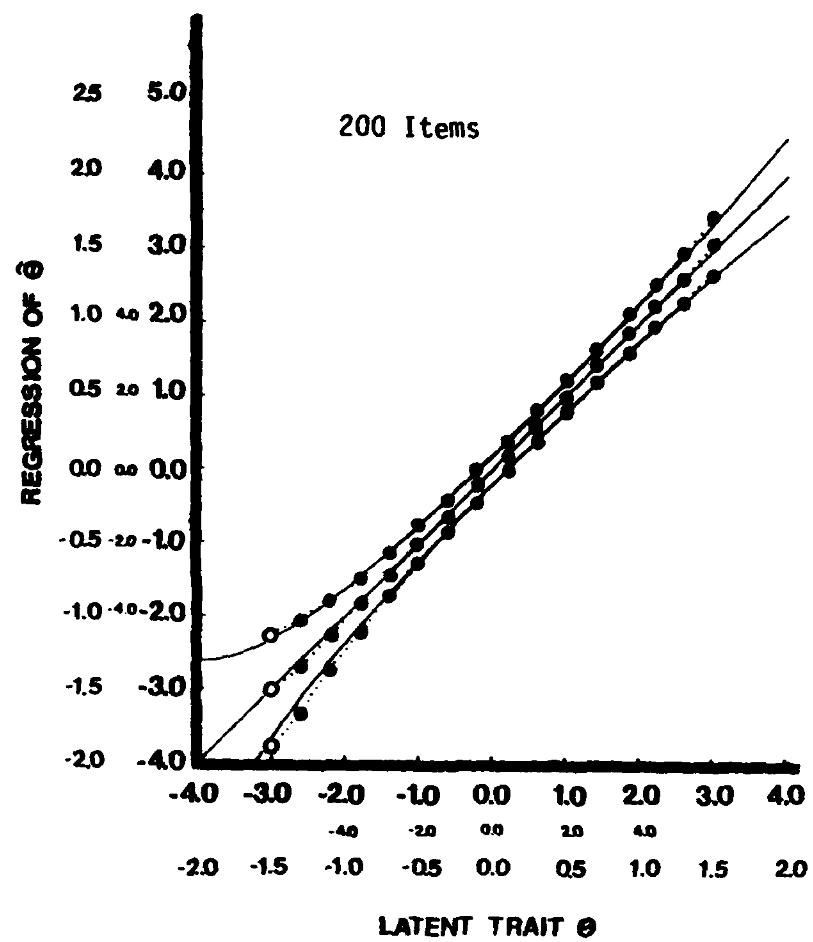


FIGURE 4-2 (Continued)

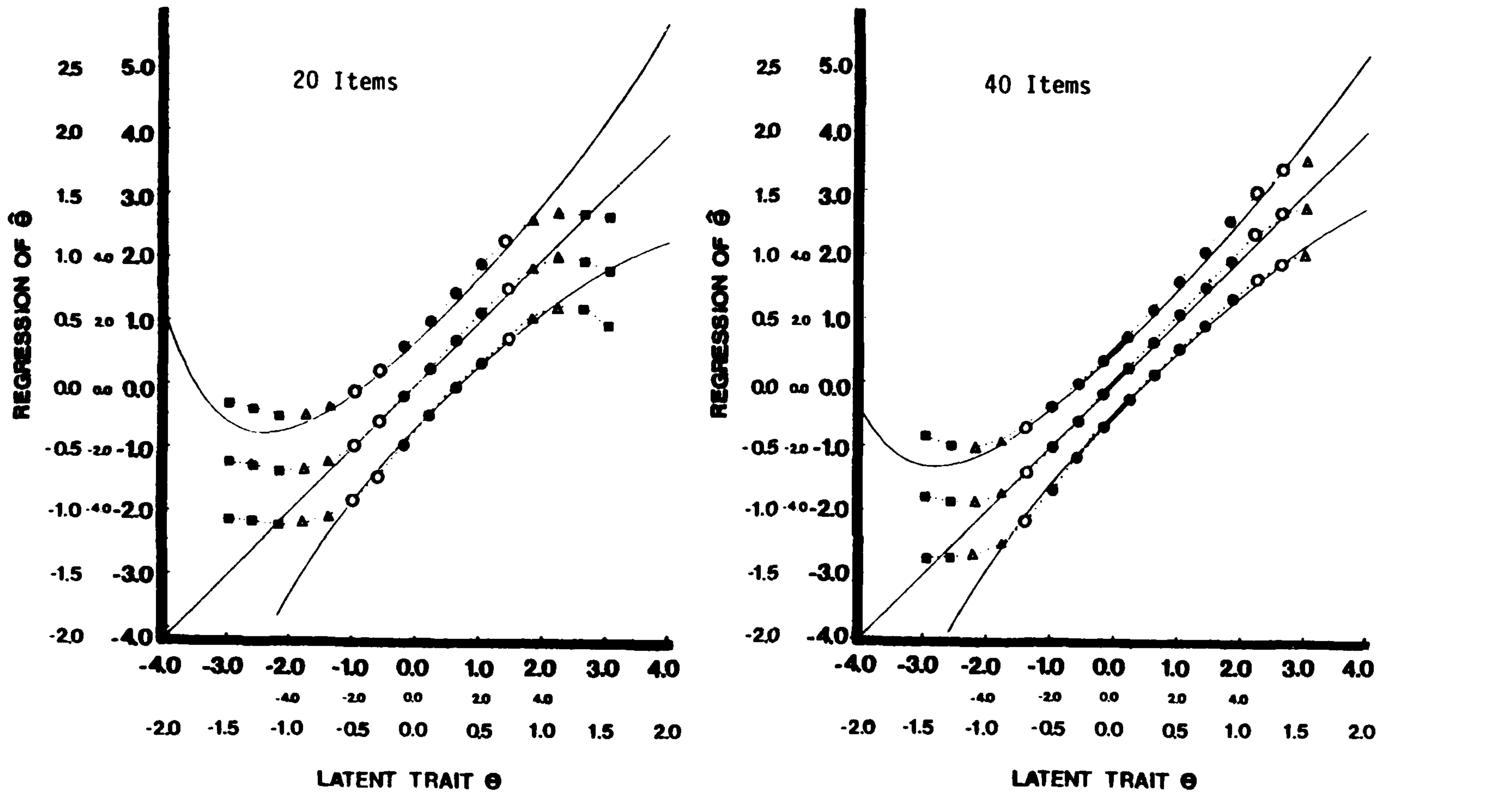


FIGURE 4-3

Regression of $\hat{\theta}_t$ on θ , μ_1' , and the Confidence Interval $(\mu_1'-\mu_2^{1/2}, \mu_1'+\mu_2^{1/2})$ Plotted against the Sixteen Points of θ , Together with the Asymptotic, Unbiased Regression and the Confidence Interval $(\theta-\{I(\theta)\}^{-1/2}, \theta+\{I(\theta)\}^{-1/2})$, for Each of the Five Tests of Equivalent Items on the Three-Parameter Logistic Model with $a_g = 0.50$, $b_g = 0.00$ and $c_g = 0.25$.

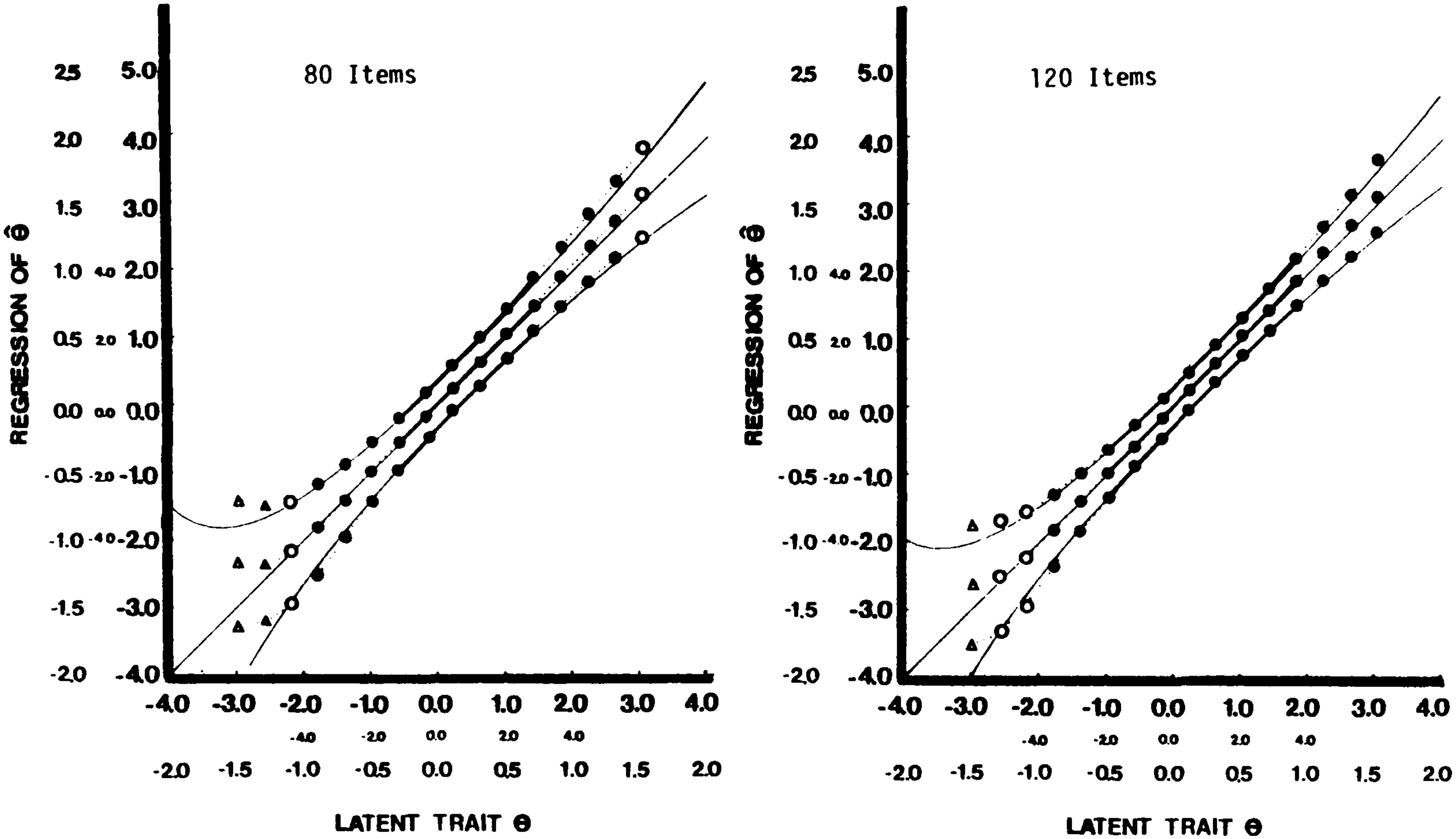


FIGURE 4-3 (Continued)

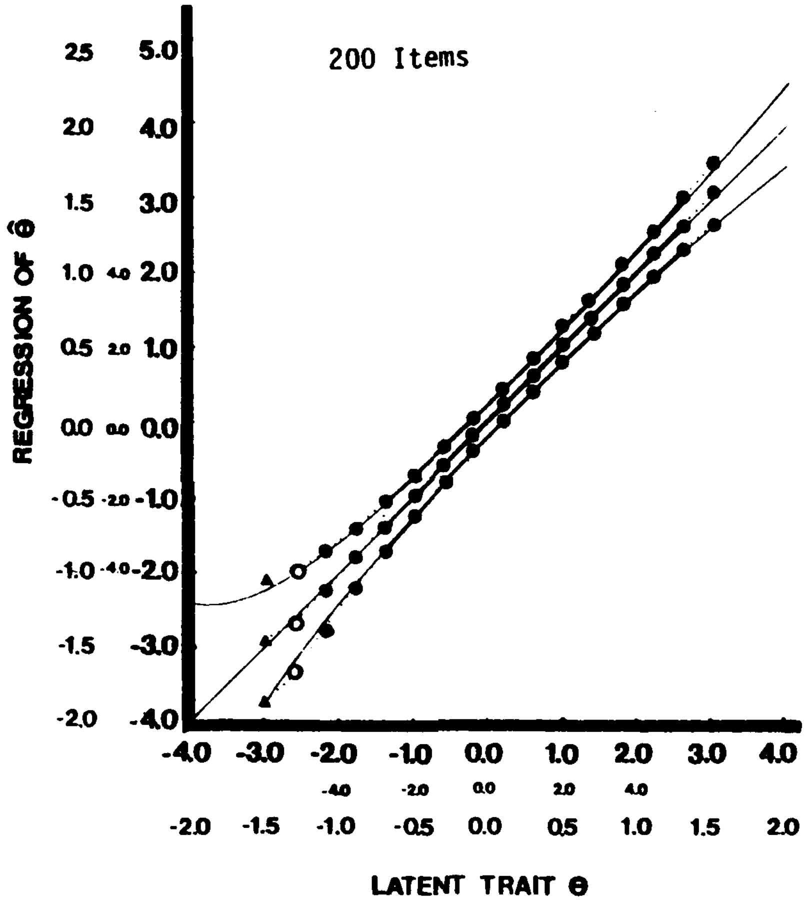


FIGURE 4-3 (Continued)

TABLE 4-6

Symbols for the Five Different Categories of the Sum of the Probabilities Assigned to Finite Values of the Maximum Likelihood Estimates.

Lower Endpoint (inclusive)	Upper Endpoint (exclusive)	Mark
----	0.80	■
0.80	0.90	△
0.90	0.95	▲
0.95	0.99	○
0.99	----	●

to, 0.999999 for two or more adjacent points of θ, the solid circles are connected by a thicker line.

We can see from those three figures that those regressions and confidence intervals are very close to the asymptotic ones, for the points of θ at which the sum total of the probabilities for finite values of $\hat{\theta}_t$ is no less than 0.99 and, especially, when the sum is greater than, or equal to, 0.999999, they practically lie on those lines. Again, we can see substantial differences with respect to the agreement between the results on the normal ogive model and those on the three-parameter logistic model.

V <u>Comparison of Tests of Non-Equivalent Items</u>

We have seen in the preceding section that the speed of convergence to the normality of the conditional distribution of the maximum likelihood estimate, given θ, is fairly high, even for a small number of equivalent items, if the values of θ are close to the point at which the amount of test information is maximal, in each of the three cases in which the normal ogive model and the three-parameter logistic model with $c_g = 0.20$ and $c_g = 0.25$ are followed. The range of θ for which this is the case is fairly small, however, for a smaller number of equivalent items, especially on the three-parameter logistic model. We notice that this interval of θ may be enhanced, if we use non-equivalent test items. For this reason, in the present section, we shall observe the regressions and confidence intervals of three hypothetical tests of ten non-equivalent items, which follow the normal ogive model with the common discrimination parameter

$a_g = 0.50$ and with the difficulty parameters b_g shown in Table 5-1 for the ten items of each of the three tests, in comparison with those of a hypothetical test of ten equivalent items which follow the same model with $a_g = 0.50$ and $b_g = 0.00$. For convenience, hereafter, we shall call any tests having the sets of difficulty parameters shown in Table 5-1, Cases 1, 2 and 3, respectively.

As we did for the five hypothetical tests of 20, 40, 80, 120 and 200 equivalent items, the results of which were shown in the preceding section, the four conditional moments, μ_1' , μ_2 , μ_3 and μ_4 , were computed by "ignoring" the probabilities assigned to the negative and positive infinities, for the hypothetical test of ten equivalent items and for the three hypothetical tests of ten non-equivalent items. Table 5-2 presents those probabilities assigned to the negative and positive infinities and their sum totals, for the four hypothetical tests of ten items. Unlike for the tests of equivalent items, for the three hypothetical tests of non-equivalent items the test score t is not a simple sufficient statistic for the response pattern V , so the maximum likelihood estimate $\hat{\theta}_V$ must be obtained as the solution of

$$(5.1) \qquad \sum_{u_g \in V} A_{u_g}(\theta) = 0 \quad ,$$

where $A_{u_g}(\theta)$ is the basic function (Samejima, 1969, 1972) defined by

$$(5.2) \qquad A_{u_g}(\theta) \begin{cases} = \frac{\partial}{\partial\theta} \log Q_g(\theta) & u_g = 0 \\ = \frac{\partial}{\partial\theta} \log P_g(\theta) & u_g = 1 \quad , \end{cases}$$

TABLE 5-1

Difficulty Parameter b_g of Each of the Ten Items in Each of the Three Cases.

Item	Case 1	Case 2	Case 3
1	-2.7	-3.6	-4.5
2	-2.1	-2.8	-3.5
3	-1.5	-2.0	-2.5
4	-0.9	-1.2	-1.5
5	-0.3	-0.4	-0.5
6	0.3	0.4	0.5
7	0.9	1.2	1.5
8	1.5	2.0	2.5
9	2.1	2.8	3.5
10	2.7	3.6	4.5

TABLE 5-2

Sum Totals of Probabilities Assigned to Negative and Positive Infinities and Their Sum in the Conditional Distribution of $\hat{\theta}_v$, given θ , at Twenty Different Values of θ , of the Four Hypothetical Tests of 10 Items Following the Normal Ogive Model. In all Four Cases, $a_g = 0.50$. Equivalent Case $b_g = 0.00$. Difficulty Parameters of the Three Non-Equivalent Cases Are Shown in Table 5-1.

Negative Infinity				Positive Infinity				Total			
Equivalent	-2.7 to 2.7	-3.6 to 3.6	-4.5 to 4.5	Equivalent	-2.7 to 2.7	-3.6 to 3.6	-4.5 to 4.5	Equivalent	-2.7 to 2.7	-3.6 to 3.6	-4.5 to 4.5
0.747240	0.426885	0.255897	0.122844	0.000000	0.000000	0.000000	0.000000	0.747240	0.426885	0.255897	0.122844
0.633883	0.310032	0.166274	0.069826	0.000000	0.000000	0.000000	0.000000	0.633883	0.310032	0.166274	0.069826
0.500857	0.205852	0.097786	0.035580	0.000000	0.000000	0.000000	0.000000	0.500857	0.205852	0.097786	0.035580
0.361274	0.122960	0.051314	0.016048	0.000000	0.000000	0.000000	0.000000	0.361274	0.122960	0.051314	0.016048
0.232710	0.064955	0.023672	0.006325	0.000000	0.000000	0.000000	0.000000	0.232710	0.064955	0.023672	0.006325
0.130792	0.029811	0.009454	0.002149	0.000000	0.000000	0.000000	0.000000	0.130792	0.029811	0.009454	0.002149
0.062647	0.011670	0.003218	0.000621	0.000001	0.000000	0.000000	0.000000	0.062648	0.011670	0.003218	0.000621
0.024985	0.003826	0.000918	0.000151	0.000008	0.000001	0.000000	0.000000	0.024993	0.003826	0.000918	0.000151
0.008115	0.001031	0.000216	0.000030	0.000066	0.000005	0.000001	0.000000	0.008181	0.001036	0.000217	0.000030
0.002102	0.000224	0.000041	0.000005	0.000426	0.000039	0.000006	0.000001	0.002527	0.000263	0.000048	0.000006
0.000426	0.000039	0.000006	0.000001	0.002102	0.000224	0.000041	0.000005	0.002527	0.000263	0.000048	0.000006
0.000066	0.000005	0.000001	0.000000	0.008115	0.001031	0.000216	0.000030	0.008181	0.001036	0.000217	0.000030
0.000008	0.000001	0.000000	0.000000	0.024985	0.003826	0.000918	0.000151	0.024993	0.003826	0.000918	0.000151
0.000001	0.000000	0.000000	0.000000	0.062647	0.011670	0.003218	0.000621	0.062648	0.011670	0.003218	0.000621
0.000000	0.000000	0.000000	0.000000	0.130792	0.029811	0.009454	0.002149	0.130792	0.029811	0.009454	0.002149
0.000000	0.000000	0.000000	0.000000	0.232710	0.064955	0.023672	0.006325	0.232710	0.064955	0.023672	0.006325
0.000000	0.000000	0.000000	0.000000	0.361274	0.122960	0.051314	0.016048	0.361274	0.122960	0.051314	0.016048
0.000000	0.000000	0.000000	0.000000	0.500857	0.205852	0.097786	0.035580	0.500857	0.205852	0.097786	0.035580
0.000000	0.000000	0.000000	0.000000	0.633883	0.310032	0.166274	0.069826	0.633883	0.310032	0.166274	0.069826
0.000000	0.000000	0.000000	0.000000	0.747240	0.426885	0.255897	0.122844	0.747240	0.426885	0.255897	0.122844

with the item characteristic function $P_g(\theta)$ replaced by the formula for that of the normal ogive model, which is given on the right hand side of (2.3), and $Q_g(\theta) = 1 - P_g(\theta)$. The conditional distribution of the maximum likelihood estimate $\hat{\theta}_V$, given θ, cannot be simplified to any form as we did for equivalent items using the probability function of the binomial distribution of the test score, for the response pattern V is not unidimensionally ordered, and for other reasons. Thus in obtaining moments we must use the operating characteristic $P_V(\theta)$ itself as the probability assigned to the maximum likelihood estimate $\hat{\theta}_V$, which is obtained from the operating characteristics of the item score through the formula

$$(5.3) \qquad P_V(\theta) = \sum_{u_g \in V} P_g(\theta)^{u_g} Q_g(\theta)^{1-u_g} \quad ,$$

provided that the distributions of the item score u_g are conditionally independent, given θ, for all the items of the test. Thus the computation of the moments is more complicated and time consuming, and we must deal with as many as 2^n different values of $\hat{\theta}_V$ instead of $(n+1)$ different values of $\hat{\theta}_t$. For this reason, we are forced to restrict our observations to tests of only ten items, in which the number of different values of $\hat{\theta}_V$ is 1,024.

Table 5-3 presents the four moments and indices β_1 and β_2, and Pearson's criterion κ and distribution type, for twenty equally spaced values of θ, -3.0 through 3.0, for each of the four tests of ten items. Just as in Tables 4-2 through 4-4, the symbol ** indicates that

Table 5-3

Conditional Mean μ_1' , Second to Fourth Moments, μ_2 , μ_3 and μ_4 , Indices β_1 and β_2 , Pearson's Criterion κ and Pearson's Type of Distribution Obtained at Sixteen Different Values of θ , for Each of the Four Hypothetical Tests of 10 Items on the Normal Ogive Model with $a_g = 0.50$ and $b_g = 0.00$. Moments Were Calculated by "Ignoring" Negative and Positive Infinities. The Mark ** Indicates That the Sum Total of the Probabilities for Finite Values of $\hat{\theta}_v$ Is Greater Than, or Equal to, 0.999999 , and * Means It Is Greater Than, or Equal to, 0.99 and Less Than 0.999999 . Equivalent Items.

		μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type	
1	-3.8	-0.6182222729D 00	0.8694376522D 00	-0.1660960145D 01	0.3198851306D 01	0.4197619715D 01	0.4231722348D 01	-0.1250115984D 01	1	
2	-3.4	-0.8707098726D 00	0.8881865769D 00	-0.1466490917D 01	0.2452655191D 01	0.3069348053D 01	0.3109052716D 01	-0.9867819727D 00	1	
3	-3.0	-0.1137990812D 01	0.7641890501D 00	-0.1055393380D 01	0.1490145140D 01	0.2495894937D 01	0.2551684349D 01	-0.8435883806D 00	1	
4	-2.6	-0.1367732347D 01	0.5875640299D 00	-0.6549460283D 00	0.8046673372D 00	0.2114683726D 01	0.2330805162D 01	-0.6564121587D 00	1	
5	-2.2	-0.1498309158D 01	0.4898849136D 00	-0.3779668683D 00	0.5460804507D 00	0.1215137067D 01	0.2275456298D 01	-0.3041421246D 00	1	
6	-1.8	-0.1482952102D 01	0.5180763556D 00	-0.1930628411D 00	0.6466038677D 00	0.2680501592D 00	0.2409077403D 01	-0.1117782452D 00	1	
7	-1.4	-0.1311362462D 01	0.6114656943D 00	-0.8701935848D-01	0.9696055551D 00	0.3312191706D-01	0.2593288906D 01	-0.2762418074D-01	1	
8	-1.0	-0.1011844542D 01	0.6920719168D 00	-0.6174497369D-01	0.1353821151D 01	0.1150136907D-01	0.2826564119D 01	-0.2270753323D-01	1	
9	-0.6	-0.6318642598D 00	0.7305546994D 00	-0.6103036805D-01	0.1669671787D 01	0.9552867668D-02	0.3128424161D 01	0.3148367785D-01	4	*
10	-0.2	-0.2140103891D 00	0.7398906506D 00	-0.2800191476D-01	0.1838049022D 01	0.1935855068D-02	0.3357546307D 01	0.2054361874D-02	4	*
11	0.2	0.2140103891D 00	0.7398906505D 00	0.2800191476D-01	0.1838049022D 01	0.1935855068D-02	0.3357546307D 01	0.2054361874D-02	4	*
12	0.6	0.6318642598D 00	0.7305546994D 00	0.6103036805D-01	0.1669671787D 01	0.9552867668D-02	0.3128424161D 01	0.3148367785D-01	4	*
13	1.0	0.1011844542D 01	0.6920719168D 00	0.6174497369D-01	0.1353821151D 01	0.1150136907D-01	0.2826564119D 01	-0.2270753323D-01	1	
14	1.4	0.1311362462D 01	0.6114656943D 00	0.8701935848D-01	0.9696055551D 00	0.3312191706D-01	0.2593288906D 01	-0.2762418074D-01	1	
15	1.8	0.1482952102D 01	0.5180763556D 00	0.1930628411D 00	0.6466038677D 00	0.2680501592D 00	0.2409077403D 01	-0.1117782452D 00	1	
16	2.2	0.1498309158D 01	0.4898849136D 00	0.3779668683D 00	0.5460804507D 00	0.1215137067D 01	0.2275456298D 01	-0.3041421246D 00	1	
17	2.6	0.1367732347D 01	0.5875640299D 00	0.6549460283D 00	0.8046673372D 00	0.2114683726D 01	0.2330805162D 01	-0.6564121587D 00	1	
18	3.0	0.1137990812D 01	0.7641890501D 00	0.1055393380D 01	0.1490145140D 01	0.2495894937D 01	0.2551684349D 01	-0.8435883806D 00	1	
19	3.4	0.8707098726D 00	0.8881865769D 00	0.1466490917D 01	0.2452655191D 01	0.3069348053D 01	0.3109052716D 01	-0.9867819727D 00	1	
20	3.8	0.6182222729D 00	0.8694376522D 00	0.1660960145D 01	0.3198851306D 01	0.4197619715D 01	0.4231722348D 01	-0.1250115984D 01	1	

Table 5-3 (Continued) Case 1.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type	
1	-3.8	-0.1777506180D 01	0.1219736220D 01	-0.2022901719D 01	0.3449204888D 01	0.2255027308D 01	0.2318393508D 01	-0.7820608726D 00	1	
2	-3.4	-0.2033018862D 01	0.9172113852D 00	-0.1245416990D 01	0.1876443612D 01	0.2010116651D 01	0.2230471325D 01	-0.6281343454D 00	1	
3	-3.0	-0.2181635768D 01	0.7517226800D 00	-0.7465901173D 00	0.1254515763D 01	0.1312174257D 01	0.2220040083D 01	-0.3289658962D 00	1	
4	-2.6	-0.2190709487D 01	0.7472214746D 00	-0.4304714213D 00	0.1292517898D 01	0.4441611965D 00	0.2314930069D 01	-0.1464091996D 00	1	
5	-2.2	-0.2054376716D 01	0.8310742686D 00	-0.2458392521D 00	0.1705533672D 01	0.1052889862D 00	0.2469338490D 01	-0.5979615574D-01	1	
6	-1.8	-0.1794038253D 01	0.9160128673D 00	-0.1848559137D 00	0.2228812646D 01	0.4445923383D-01	0.2656258974D 01	-0.4129024484D-01	1	
7	-1.4	-0.1447233337D 01	0.9575445297D 00	-0.1938203256D 00	0.2661679922D 01	0.4278797162D-01	0.2902938729D 01	-0.1006506596D 00	1	
8	-1.0	-0.1051925295D 01	0.9566691148D 00	-0.1920368818D 00	0.2890883027D 01	0.4211957989D-01	0.3158690051D 01	0.1671540807D 00	4	*
9	-0.6	-0.6354503078D 00	0.9364888808D 00	-0.1408142523D 00	0.2926655661D 01	0.2414267295D-01	0.3337078[illegible]1D 01	0.3034167787D-01	4	*
10	-0.2	-0.2122325534D 00	0.9201365218D 00	-0.5109309849D-01	0.2888294980D 01	0.3350947894D-02	0.3411434267D 01	0.3107055833D-02	4	*
11	0.2	0.2122325534D 00	0.9201365218D 00	0.5109309849D-01	0.2888294980D 01	0.3350947894D-02	0.3411434267D 01	0.3107055833D-02	4	*
12	0.6	0.6354503078D 00	0.9364888808D 00	0.1408142523D 00	0.2926655661D 01	0.2414267295D-01	0.3337078151D 01	0.3034167787D-01	4	*
13	1.0	0.1051925295D 01	0.9566691148D 00	0.1920368818D 00	0.2890883027D 01	0.4211957989D-01	0.3158690051D 01	0.1671540807D 00	4	*
14	1.4	0.1447233337D 01	0.9575445297D 00	0.1938203256D 00	0.2661679922D 01	0.4278797162D-01	0.2902938729D 01	-0.1006506596D 00	1	
15	1.8	0.1794038253D 01	0.9160128673D 00	0.1848559137D 00	0.2228812646D 01	0.4445923383D-01	0.2656258974D 01	-0.4129024484D-01	1	
16	2.2	0.2054376716D 01	0.8310742686D 00	0.2458392521D 00	0.1705533672D 01	0.1052889862D 00	0.2469338490D 01	-0.5979615574D-01	1	
17	2.6	0.2190709487D 01	0.7472214746D 00	0.4304714213D 00	0.1292517898D 01	0.4441611965D 00	0.2314930069D 01	-0.1464091996D 00	1	
18	3.0	0.2181635768D 01	0.7517226800D 00	0.7465901173D 00	0.1254515763D 01	0.1312174257D 01	0.2220040083D 01	-0.3289658962D 00	1	
19	3.4	0.2033018862D 01	0.9172113852D 00	0.1245416990D 01	0.1876443612D 01	0.2010116651D 01	0.2230471325D 01	-0.6281343454D 00	1	
20	3.8	0.1777506180D 01	0.1219736220D 01	0.2022901719D 01	0.3449204888D 01	0.225502[illegible]8D 01	0.[illegible]8393508D 01	-0.7820608726D 00	1	

Table 5-3 (Continued) Case 2.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type
1	-3.8	-0.2533940348D 01	0.1042747353D 01	-0.1427595221D 01	0.2332965802D 01	0.1797516487D 01	0.2145607015D 01	-0.5252571089D 00	1
2	-3.4	-0.2656699650D 01	0.8935454352D 00	-0.8495847701D 00	0.1738179762D 01	0.1011727409D 01	0.2177014933D 01	-0.2552737610D 00	1
3	-3.0	-0.2634252412D 01	0.9140923819D 00	-0.4866397269D 00	0.1929056700D 01	0.3100593634D 00	0.2308685705D 01	-0.1137372387D 00	1
4	-2.6	-0.2471964635D 01	0.1007711310D 01	-0.2873033795D 00	0.2508834530D 01	0.8066275861D-01	0.2470584730D 01	-0.4812502145D-01	1
5	-2.2	-0.2196809920D 01	0.1089745358D 01	-0.2303965091D 00	0.3154349543D 01	0.4101821017D-01	0.2656193819D 01	-0.3853561806D-01	1
6	-1.8	-0.1845698126D 01	0.1124496962D 01	-0.2455452423D 00	0.3643913156D 01	0.4240218021D-01	0.2881717772D 01	-0.8843341928D-01	1 *
7	-1.4	-0.1452769997D 01	0.1116686311D 01	-0.2500057884D 00	0.3864303724D 01	0.4488554998D-01	0.3098909325D 01	0.5389765076D 00	4 *
8	-1.0	-0.1041814025D 01	0.1088117379D 01	-0.2087836783D 00	0.3836452756D 01	0.3383497418D-01	0.3240248821D 01	0.6758573599D-01	4 *
9	-0.6	-0.6254491296D 00	0.1059412291D 01	-0.1333059949D 00	0.3692626247D 01	0.1494528973D-01	0.3290071511D 01	0.2105567965D-01	4 *
10	-0.2	-0.2084322761D 00	0.1042724480D 01	-0.4510167246D-01	0.3579155192D 01	0.1794223680D-02	0.3291860254D 01	0.2332755980D-02	4 *
11	0.2	0.2084322761D 00	0.1042724480D 01	0.4510167246D-01	0.3579155192D 01	0.1794223680D-02	0.3291860254D 01	0.2332755980D-02	4 *
12	0.6	0.6254491296D 00	0.1059412291D 01	0.1333059949D 00	0.3692626247D 01	0.1494528973D-01	0.3290071511D 01	0.2105567965D-01	4 *
13	1.0	0.1041814025D 01	0.1088117379D 01	0.2087836783D 00	0.3836452756D 01	0.3383497418D-01	0.3240248821D 01	0.6758573599D-01	4 *
14	1.4	0.1452769997D 01	0.1116686311D 01	0.2500057884D 00	0.3864303724D 01	0.4488554998D-01	0.3098909325D 01	0.5389765076D 00	4 *
15	1.8	0.1845698126D 01	0.1124496962D 01	0.2455452423D 00	0.3643913156D 01	0.4240218021D-01	0.2881717772D 01	-0.8843341928D-01	1 *
16	2.2	0.2196809920D 01	0.1089745358D 01	0.2303965091D 00	0.3154349543D 01	0.4101821017D-01	0.2656193819D 01	-0.3853561806D-01	1
17	2.6	0.2471964635D 01	0.1007711310D 01	0.2873033795D 00	0.2508834530D 01	0.8066275861D-01	0.2470584730D 01	-0.4812502145D-01	1
18	3.0	0.2634252412D 01	0.9140923819D 00	0.4866397269D 00	0.1929056700D 01	0.3100593634D 00	0.2308685705D 01	-0.1137372387D 00	1
19	3.4	0.2656699650D 01	0.8935454352D 00	0.8495847701D 00	0.1738179762D 01	0.1011727409D 01	0.2177014933D 01	-0.2552737610D 00	1
20	3.8	0.2533940348D 01	0.1042747353D 01	0.1427595221D 01	0.2332965802D 01	0.1797516487D 01	0.2145607015D 01	-0.5252571089D 00	1

Table 5-3 (Continued) Case 3.

	θ	μ_1'	μ_2	μ_3	μ_4	β_1	β_2	κ	type
1	-3.8	-0.3193994263D 01	0.1051862652D 01	-0.8497321011D 00	0.2413753059D 01	0.6204216505D 00	0.2181598200D 01	-0.1734113320D 00	1
2	-3.4	-0.3116403752D 01	0.1114944930D 01	-0.4760283755D 00	0.2918872941D 01	0.1634953597D 00	0.2348055490D 01	-0.7318958947D-01	1
3	-3.0	-0.2909482264D 01	0.1217592014D 01	-0.2946341922D 00	0.3730972025D 01	0.4809067992D-01	0.2516624445D 01	-0.3319063440D-01	1
4	-2.6	-0.2605907571D 01	0.1290008506D 01	-0.2592348268D 00	0.4499123120D 01	0.3130465850D-01	0.2703601819D 01	-0.3458270993D-01	1
5	-2.2	-0.2241445500D 01	0.1312698895D 01	-0.2796290193D 00	0.5009668049D 01	0.3456759140D-01	0.2907225439D 01	-0.9045895605D-01	1 *
6	-1.8	-0.1845470715D 01	0.1297352475D 01	-0.2797812728D 00	0.5182180124D 01	0.3584786781D-01	0.3078906746D 01	0.5396334812D 00	4 *
7	-1.4	-0.1436956139D 01	0.1264978345D 01	-0.2382591021D 00	0.5078784690D 01	0.2804463093D-01	0.3173902782D 01	0.8036739266D-01	4 *
8	-1.0	-0.1025488377D 01	0.1232436057D 01	-0.1714661121D 00	0.4850024663D 01	0.1570592117D-01	0.3193119377D 01	0.3489776266D-01	4 *
9	-0.6	-0.6145711368D 00	0.1208594186D 01	-0.9995338218D-01	0.4636386293D 01	0.5659179723D-02	0.3174085433D 01	0.1284279977D-01	4 *
10	-0.2	-0.2047109830D 00	0.1196433364D 01	-0.3248261547D-01	0.4516188392D 01	0.6160790438D-03	0.3154968444D 01	0.1500931952D-02	4 *
11	0.2	0.2047109830D 00	0.1196433364D 01	0.3248261547D-01	0.4516188392D 01	0.6160790438D-03	0.3154968444D 01	0.1500931952D-02	4 *
12	0.6	0.6145711368D 00	0.1208594186D 01	0.9995338218D-01	0.4636386293D 01	0.5659179723D-02	0.3174085433D 01	0.1284279977D-01	4 *
13	1.0	0.1025488377D 01	0.1232436057D 01	0.1714661121D 00	0.4850024663D 01	0.1570592117D-01	0.3193119377D 01	0.3489776266D-01	4 *
14	1.4	0.1436956139D 01	0.1264978345D 01	0.2382591021D 00	0.5078784690D 01	0.2804463093D-01	0.3173902782D 01	0.8036739266D-01	4 *
15	1.8	0.1845470715D 01	0.1297352475D 01	0.2797812728D 00	0.5182180124D 01	0.3584786781D-01	0.3078906746D 01	0.5396334812D 00	4 *
16	2.2	0.2241445500D 01	0.1312698895D 01	0.2796290193D 00	0.5009668049D 01	0.3456759140D-01	0.2907225439D 01	-0.9045895605D-01	1 *
17	2.6	0.2605907571D 01	0.1290008506D 01	0.2592348268D 00	0.4499123120D 01	0.3130465850D-01	0.2703601819D 01	-0.3458270993D-01	1
18	3.0	0.2909482264D 01	0.1217592014D 01	0.2946341922D 00	0.3730972025D 01	0.4809067992D-01	0.2516624445D 01	-0.3319063440D-01	1
19	3.4	0.3116403752D 01	0.1114944930D 01	0.4760283755D 00	0.2918872941D 01	0.1634953597D 00	0.2348055490D 01	-0.7318958947D-01	1
20	3.8	0.3193994263D 01	0.1051862652D 01	0.8497321011D 00	0.2413753059D 01	0.6204216505D 00	0.2181598200D 01	-0.1734113320D 00	1

the sum total of the probabilities assigned to the finite values of $\hat{\theta}_V$ is greater than, or equal to, 0.999999 , and * means that it is greater than, or equal to, 0.99 , but less than 0.999999 . We can see that none of the rows of the table are marked with ** , unlike the other cases in which the number of items is 20 or greater, which we observed in the preceding section. Comparison of the results of the three non-equivalent cases with those of the equivalent case reveals that, around $\theta = 0.00$, which equals the common difficulty parameter b_g for the equivalent case and the mean of the difficulty parameters for each of the three non-equivalent cases, the conditional distribution of $\hat{\theta}_V$, given θ , is closer to the normality for a wider range of θ , as the difficulty parameters spread more widely. If we take the arbitrary criterion as we did in the preceding section, i.e., $|\mu_1'-\theta| < 0.03$, $\beta_1 < 0.05$, $|\beta_2-3| < 0.50$ and $|\kappa| < 0.25$, then this criterion is satisfied for the six values of θ , -1.0 , -0.6 , -0.2 , 0.2 , 0.6 and 1.0 , for Case 3, while this number reduces to four in Case 2, and two for Case 1 and for the equivalent case.

Figure 5-1 presents the two kinds of regression plus confidence interval for each of the four cases, which were observed in the preceding section for each of the five equivalent tests on the normal ogive and on the three-parameter logistic models. The two additional sets of numbers shown on both the abscissa and the ordinate are the same scale changes for $a_g = 2.00$ and $a_g = 1.00$, that are shown in Figures 4-2 through 4-4 . Note, however, following each scale change, the difficulty parameters in the three non-equivalent cases are also changed proportionally. We can

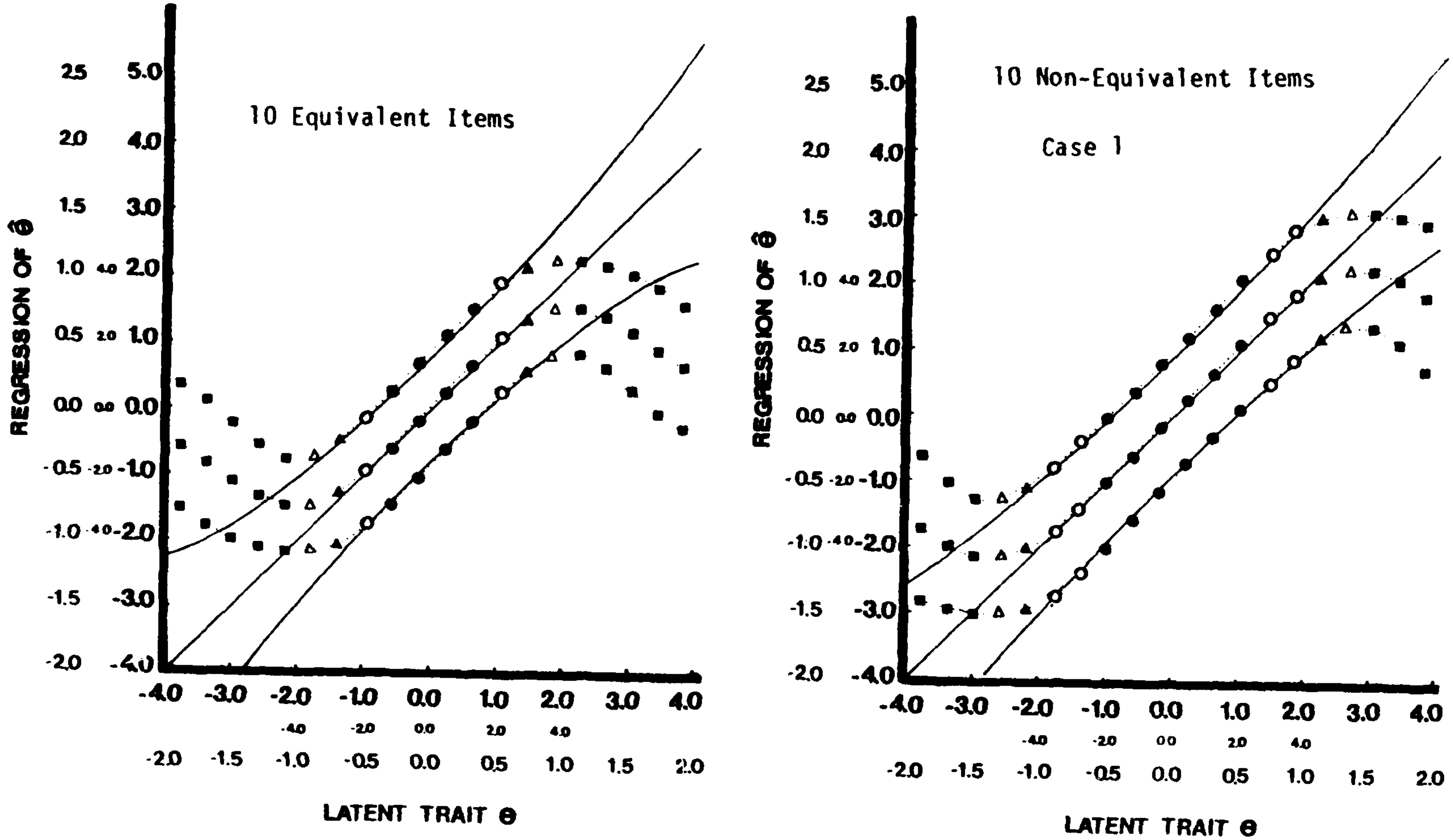


FIGURE 5-1

Regression of $\hat{\theta}_v$ on θ , or μ_1' , and the Confidence Interval $(\mu_1'-\mu^{1/2}, \mu_1'+\mu_2^{1/2})$ Plotted Against the Twenty Points of θ , Together with the Asymptotic, Unbiased Regression and the Confidence Interval $(\theta-\{I(\theta)\}^{-1/2}, \theta+\{I(\theta)\}^{-1/2})$, for Each of the Four Tests of 10 Items on the Normal Ogive Model with $a_g = 0.50$. For the Equivalent Case, $b_g = 0.00$, and the Three Sets of Difficulty Parameters for the Three Non-Equivalent Cases Are as Shown in Table 5-1.

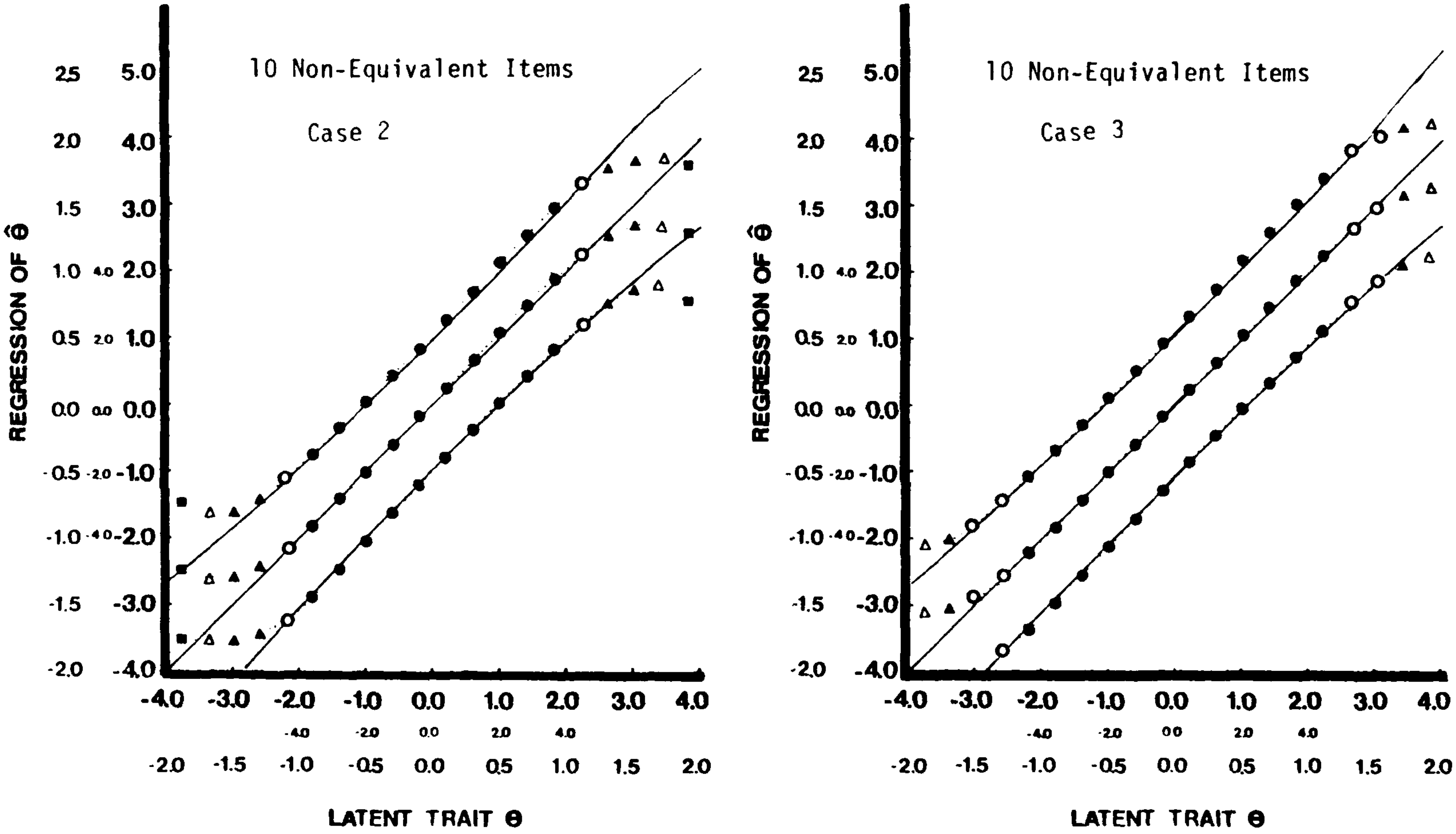


FIGURE 5-1 (Continued)

see that in each case, those solid circles, which are based upon the sum total of the probabilities assigned to the finite values of $\hat{\theta}_V$ greater than, or equal to, 0.99 , practically lie on the asymptotic regression and the confidence interval, i.e., θ and $(\theta-\{I(\theta)\}^{-1/2}, \theta+\{I(\theta)\}^{-1/2})$. There are substantial differences in the range of θ for which this is the case, however. This interval of θ is (-2.2, 2.2) for Case 3 of the non-equivalent items, while it is only (-0.6 ,0.6) for the equivalent case.

The square root of the test information function of each of the three non-equivalent cases is shown by a dashed line in each graph, in both Figures 5-2 and 5-3. In the same figures, also presented by solid lines are the square roots of the test information functions of the ten item tests which are based upon the three-parameter logistic model with the same parameters a_g and b_g and the third parameter, $c_g = 0.20$ and $c_g = 0.25$, respectively. The critical values, θ_g , of the first items of the three tests are -0.36467282900004818D 01 , -0.45467282900004819D 01 and -0.54467282900004819D 01 for $c_g = 0.20$, and -0.35154673627399690D 01 , -0.44154673627399690D 01 and -0.53154673627399690D 01 for $c_g = 0.25$. In Case 1, the values of θ_g increase by 0.6 as the value of the difficulty parameter increases, while in Cases 2 and 3 the steps are 0.8 and 1.0 , respectively.

If we take the strategy not to use the informaton obtained from item g for the interval of θ , $(-\infty, \theta_g)$, then the square root of the test information function will be reduced to the curve plotted by a dotted line in each of the six graphs of Figures 5-2 and 5-3. In this way, we can

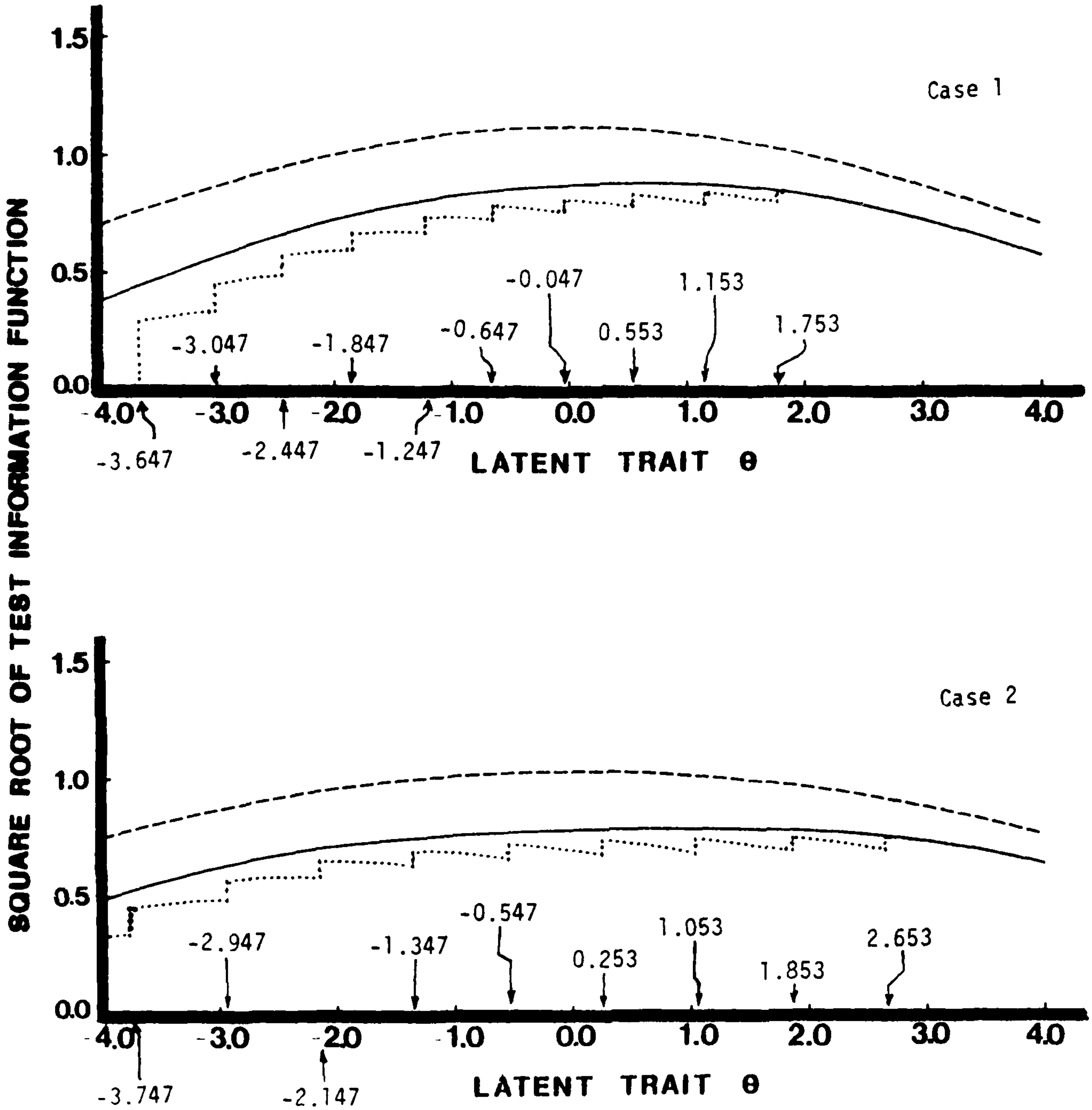


FIGURE 5-2

Square Roots of Test Information Functions of Each of the Three Tests of Ten Non-Equivalent Items in the Normal Ogive Model (Dashed Line) and in the Three-Parameter Logistic Model (Solid Line). The Latter Is Reduced to the One Drawn by Dots When Each Item Information Function Is Truncated at θ_g , Whose Value Is Shown in Each Graph.

$c_g = 0.20$

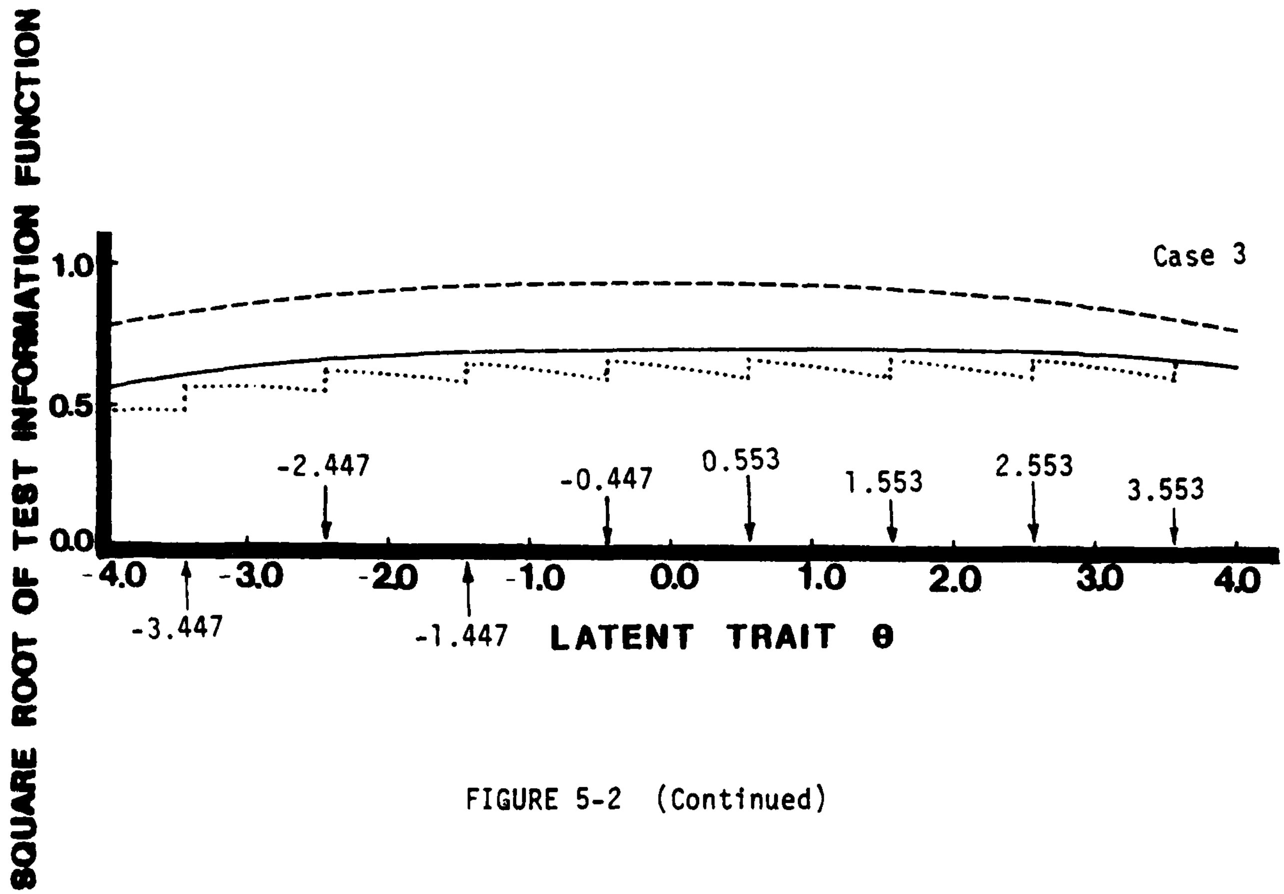


FIGURE 5-2 (Continued)

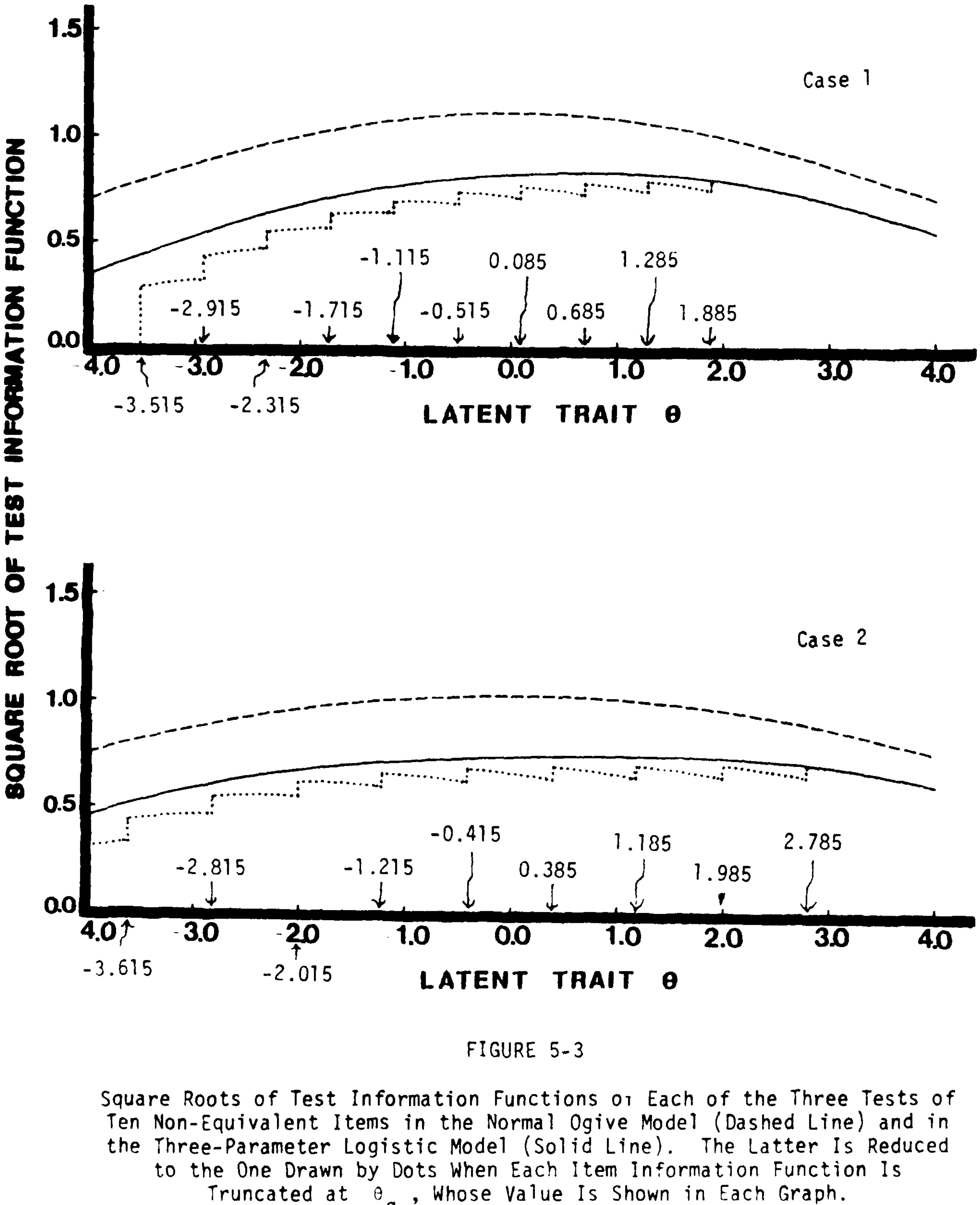


FIGURE 5-3

Square Roots of Test Information Functions of Each of the Three Tests of Ten Non-Equivalent Items in the Normal Ogive Model (Dashed Line) and in the Three-Parameter Logistic Model (Solid Line). The Latter Is Reduced to the One Drawn by Dots When Each Item Information Function Is Truncated at θ_g, Whose Value Is Shown in Each Graph.
$c_g = 0.25$

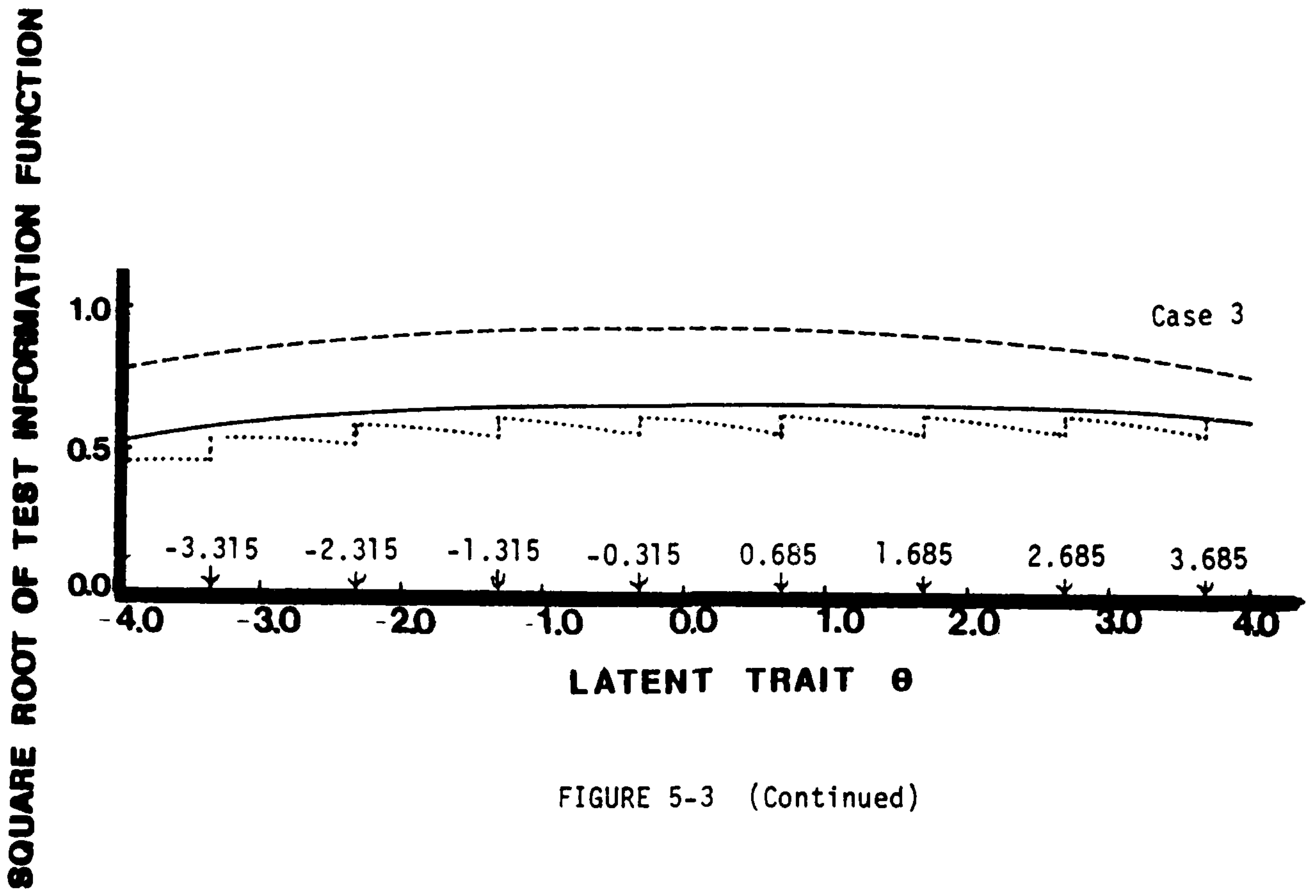


FIGURE 5-3 (Continued)

avoid multi-modal likelihood functions which may occur for some response patterns if the items follow the three-parameter logistic model. Further investigation as to the merit and demerit of this strategy is yet to come, however, and its results are left to another paper. The corresponding six graphs of the test information functions are presented as Figures A-2 and A-3 in Appendix.

VI Discussion and Conclusions

The three-parameter logistic model was compared with the normal ogive model using hypothetical tests of equivalent items, mainly with respect to the speed of convergence of the conditional distribution of the maximum likelihood estimate, given θ, to the normality, and it was found out that the effect of noise caused by random guessing is substantial, especially for the values of θ less than the critical value θ_g. Some observations were made on the normal ogive model as to how the interval of θ for which the approximation of the conditional distribution by the normality is acceptable can be enhanced by using non-equivalent test items.

This is just a beginning of the investigation about how and in what ways we can amend the deficiencies of the three-parameter logistic model which are caused by random guessing. The effective use of the critical value θ_g may be one solution to the problem, which will be investigated further.

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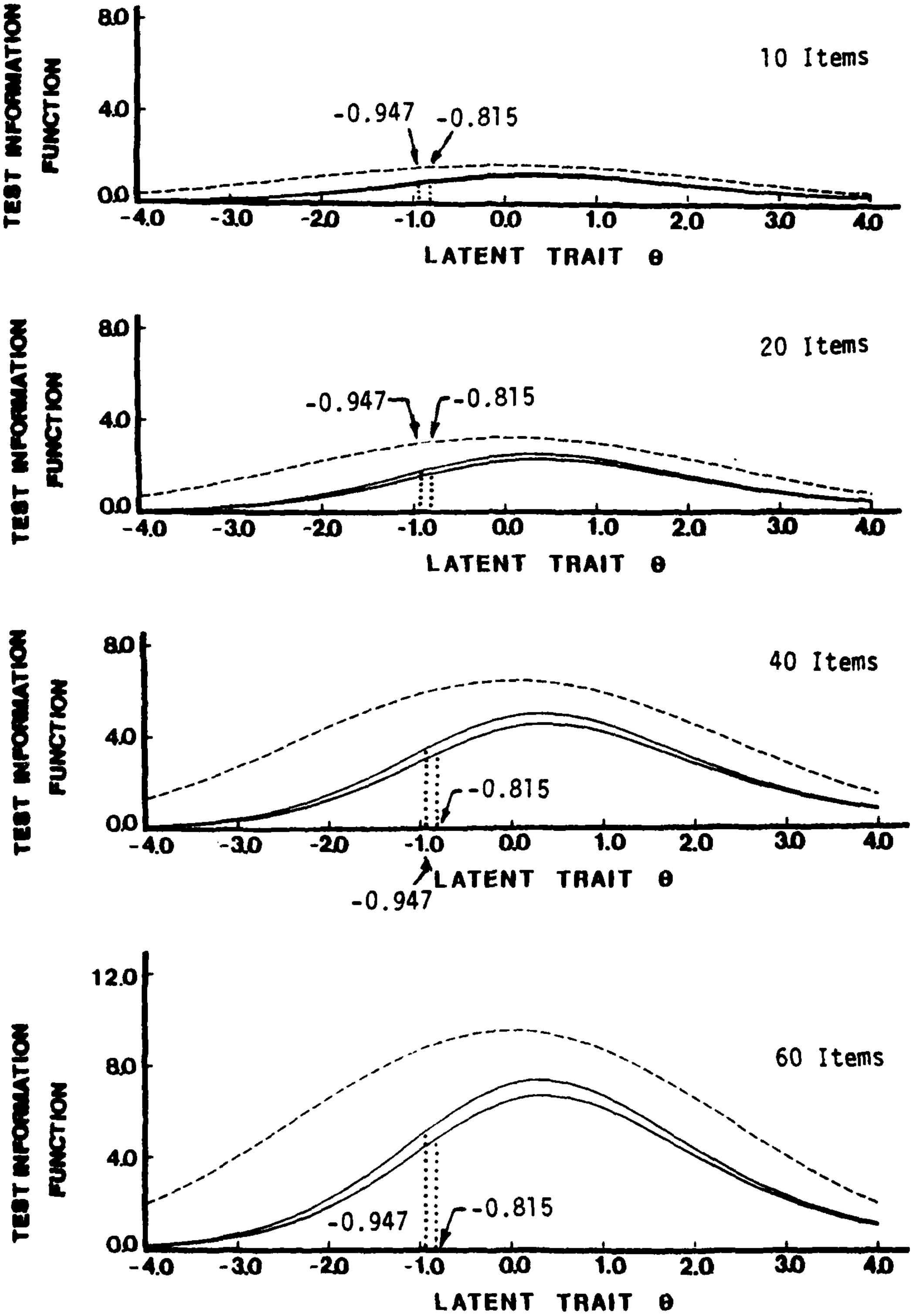
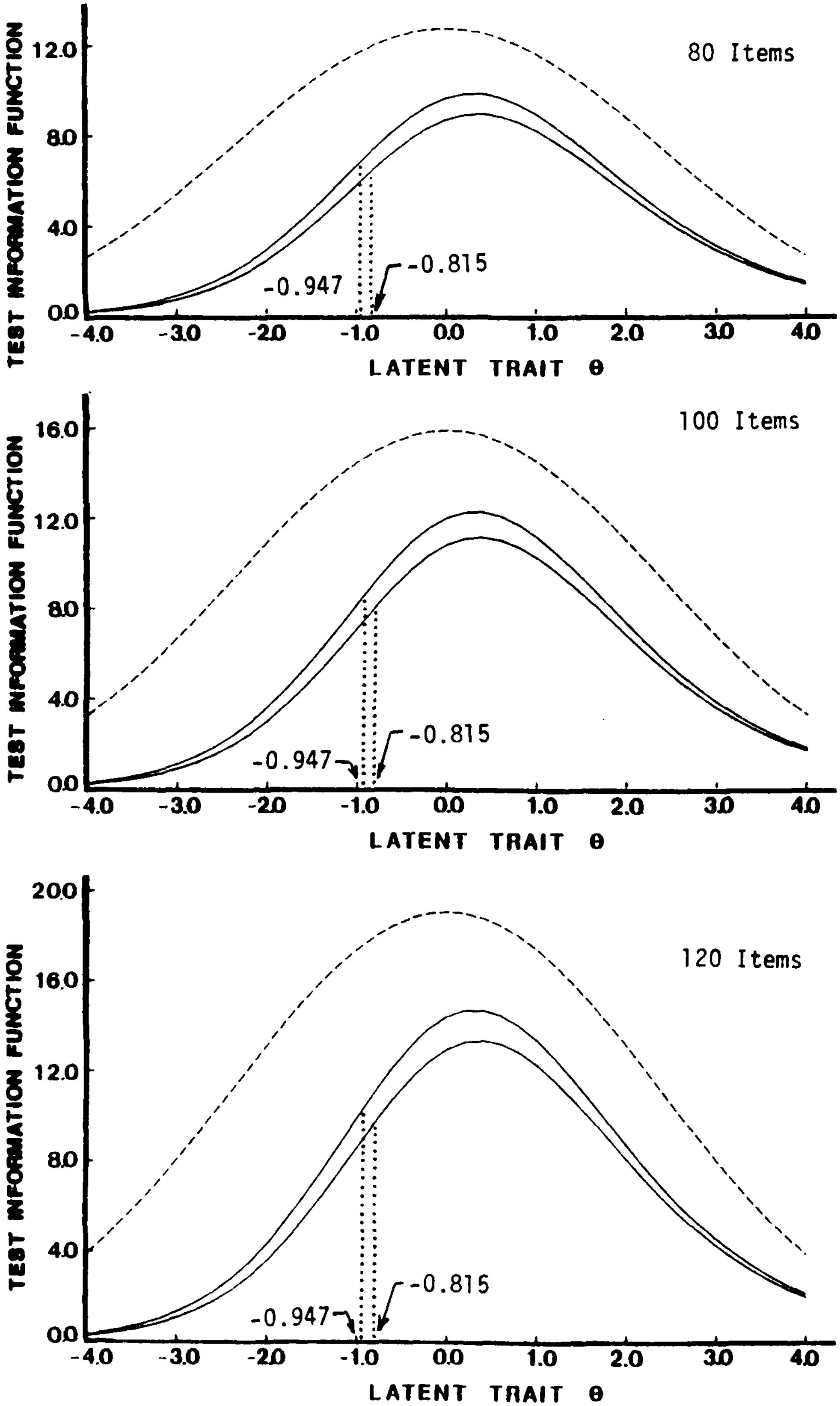


FIGURE A-1

Test Information Functions of Each of Eleven Tests of Equivalent Items, in the Normal Ogive Model (Dashed Line), and in the Three-Parameter Logistic Model (Solid Lines) with $c_g = 0.20$ and $c_g = 0.25$. The Two Values of the Common θ_g Are Shown.



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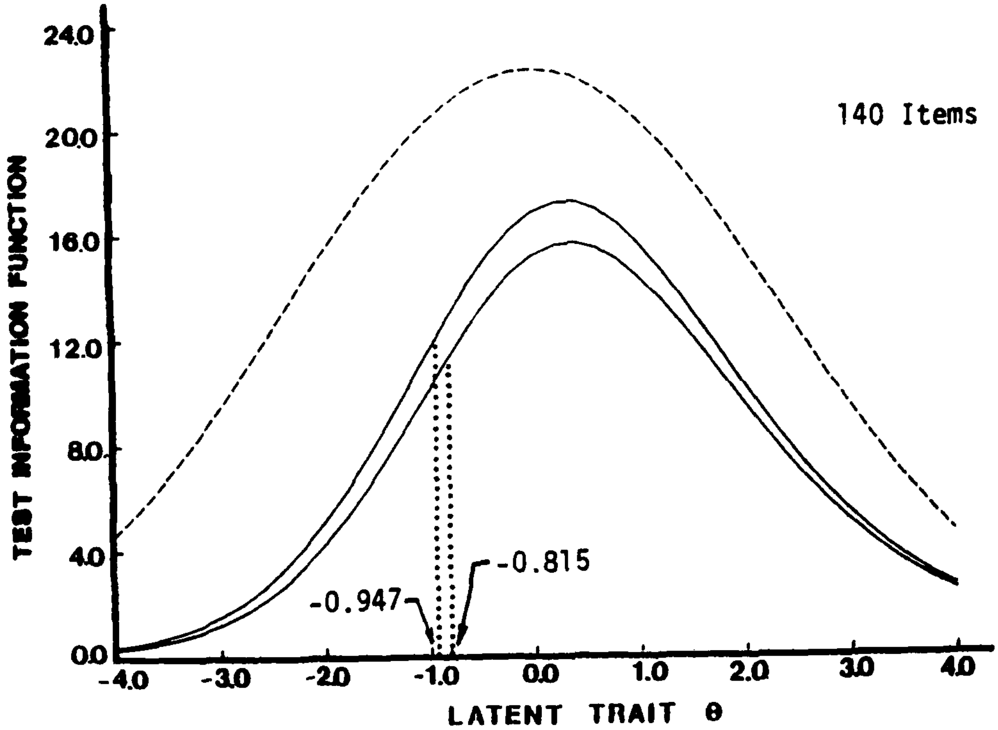

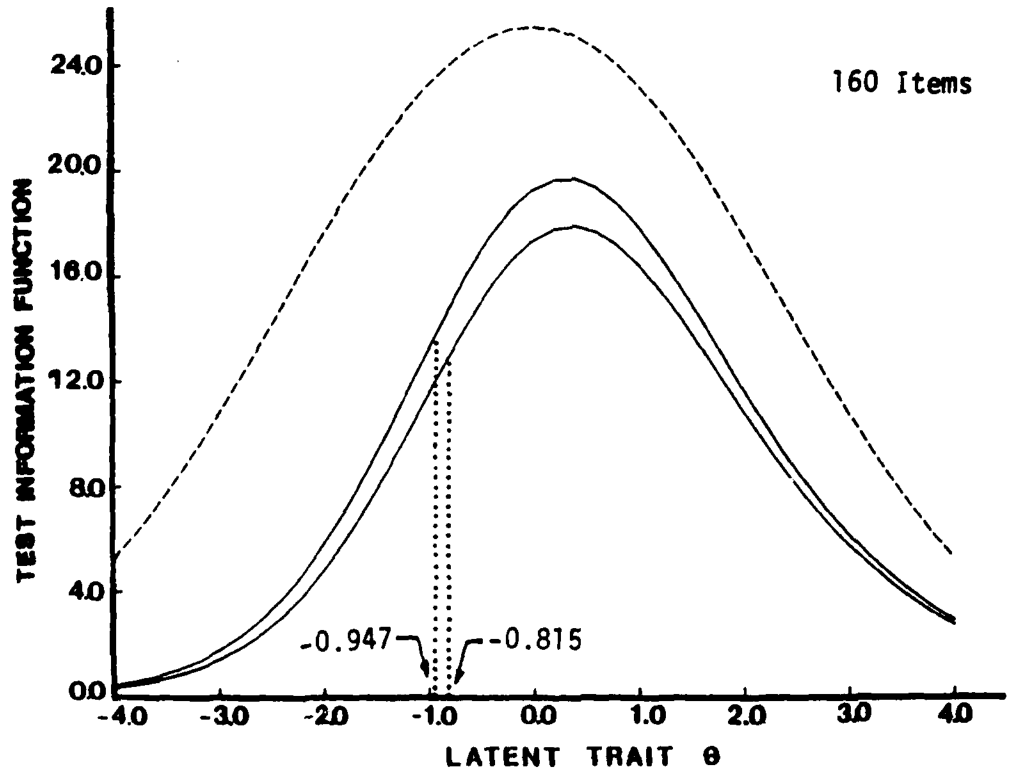

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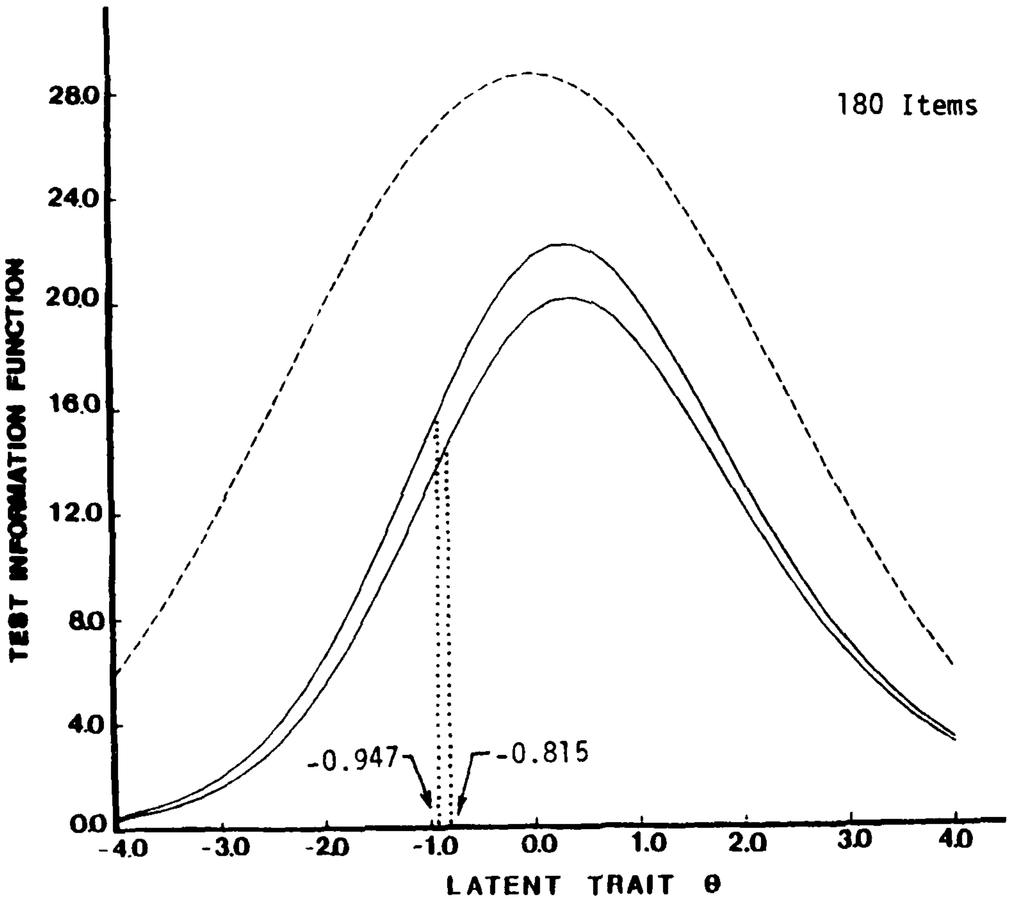

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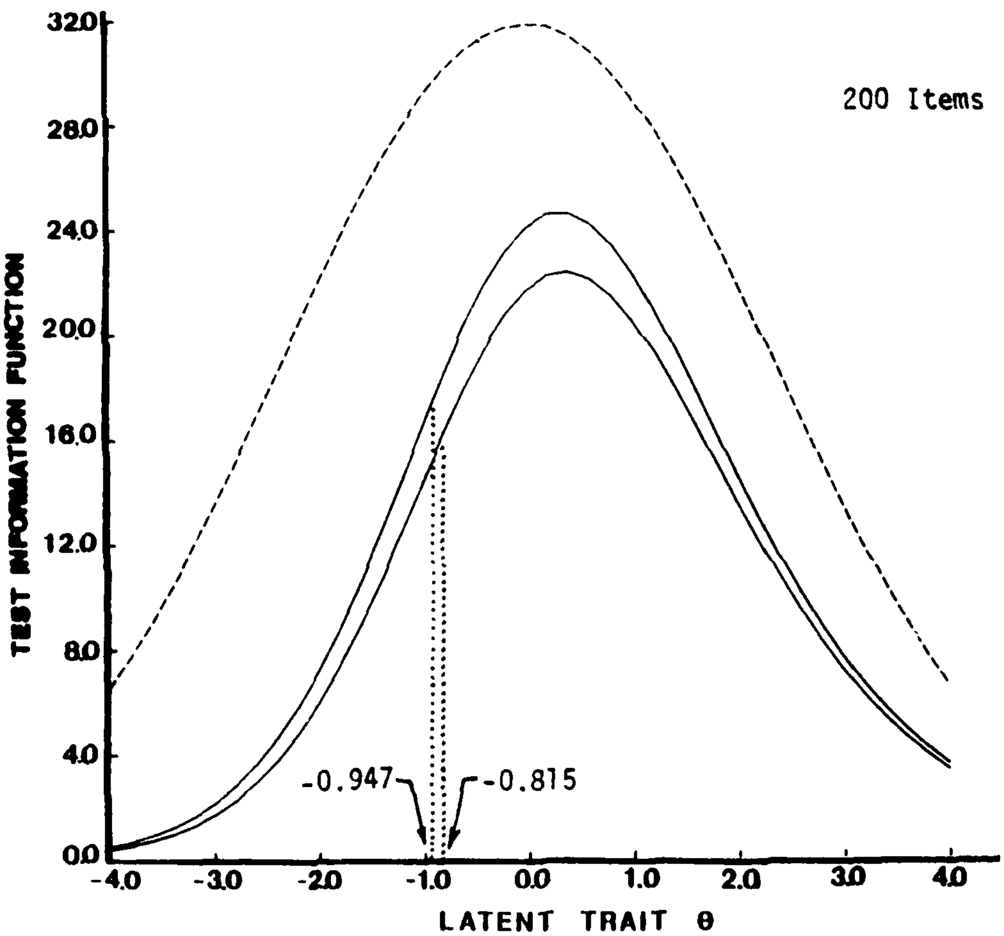

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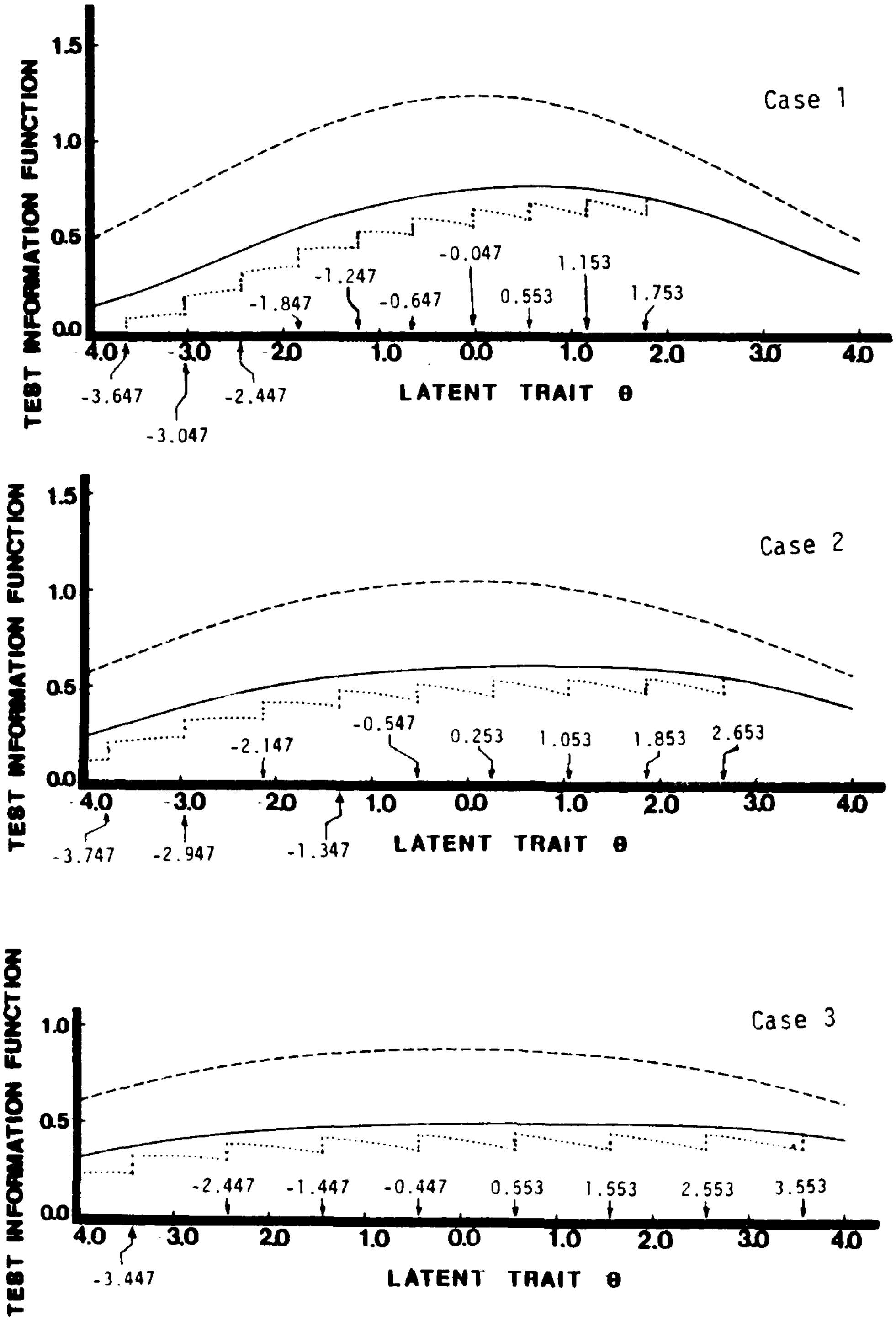

FIGURE A-2

Test Information Functions of Each of the Three Tests of Ten Non-Equivalent Items in the Normal Ogive Model (Dashed Line) and in the Three-Parameter Logistic Model (Solid Line). The Latter Is Reduced to the One Drawn by Dots When Each Item Information Function Is Truncated at θ_g, Whose Value Is Shown in Each Graph.

$c_g = 0.20$

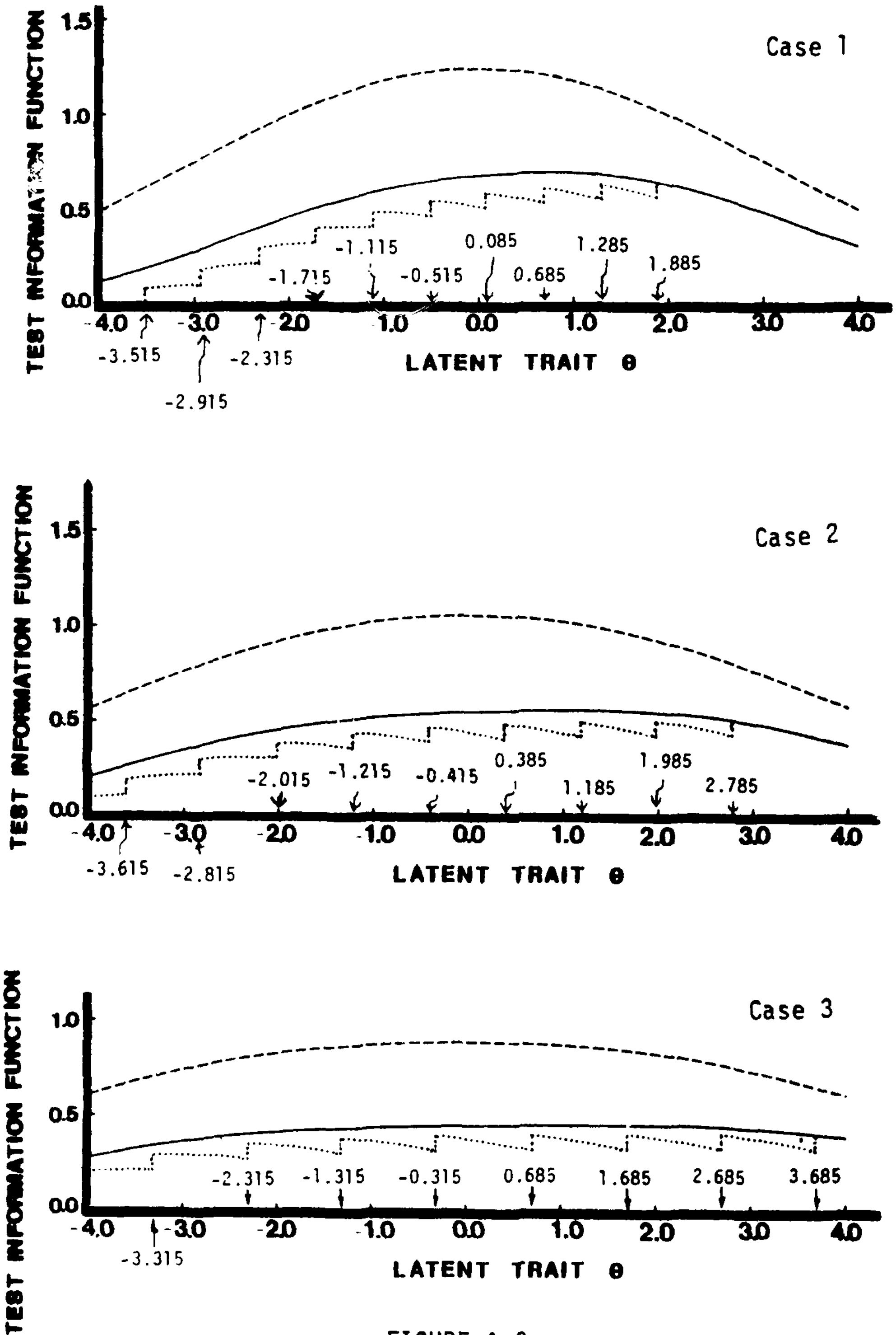

FIGURE A-3

Test Information Functions of Each of the Three Tests of Ten Non-Equivalent Items in the Normal Ogive Model (Dashed Line) and in the Three-Parameter Logistic Model (Solid Line). The Latter Is Reduced to the One Drawn by Dots When Each Item Information Function Is Truncated at θ_g, Whose Value Is Shown in Each Graph.

$c_g = 0.25$

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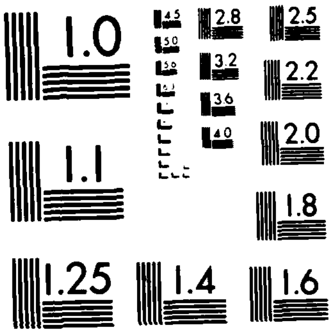

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